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A SHORT TABLE OF LANCHESTER-CLIFFORD-SCHLAFLI FUNCTIONS.(U)

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NAVAL POSTGRADUATE SCHOOL

Monterey, California

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LANCHESTER-CLIFFORD-SCHLÄFLI FUNCTIONS

by

James G. Taylor

and

Gerald G. Brown

October 1977

NAVAL POSTGRADUATE SCHOOL
Monterey, California

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This report contains a reduced set of tables of Lanchester-Clifford-Schläfli (LCS) functions. A companion report contains a more extensive (and currently the most extensive available) set of tables of the LCS functions. These functions may be used to analyze Lanchester-type combat between two homogeneous forces modelled by power attrition-rate coefficients with "no effect." Theoretical background for the LCS functions is given, as well as a narrative description of the physical circumstances under which the associated

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Lanchester-type combat model may be expected to be applicable. Numerical examples are given to illustrate the use of the LCS functions for analyzing "aimed-fire" combat modelled by the power attrition-rate coefficients with "no offset." Our results and these tabulations allow one to study this particular variable-coefficient combat model almost as easily and thoroughly as Lanchester's classic constant-coefficient model.

A SHORT TABLE OF LANCHESTER-CLIFFORD-SCHLÄFLI FUNCTIONS

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TABLE OF CONTENTS

1. Introduction

Lanchester-type* differential-equation combat models are an important tool for analyzing many important problems of military operations research. In such a combat model, a so-called *attrition-rate coefficient represents* the fire effectiveness of a particular weapon-system type against a particular target type, i.e. the weapon-system type's effective firepower against such a target. Time-dependent attrition-rate coefficients are used to model temporal variations in firepower on the battlefield. Thus, we see that time-dependent attrition-rate coefficients are important (and, in fact, essential [4-6]) for the quantitative analysis of hypothetical combat.

Militarily realistic computer-based Lanchester-type models of quite complex military systems have been developed for almost the entire spectrum of combat operations, from combat between battalion-sized units to theater-level operations. Nevertheless, a simple combat model may yield a clearer understanding of significant interrelationships that are difficult to perceive in a more complex model, and such insights can subsequently provide valuable guidance for more detailed computerized investigations. In this report we consider such a simplified variable-coefficient Lanchester-type model of combat between two homogeneous forces.

For this variable-coefficient Lanchester-type model of combat between two homogeneous forces, different functional forms for the attrition-rate coefficients lead to different mathematical functions being involved in representing and computing the force-level trajectories. In a previous paper [5] we have discussed the plausibility of the hypothesis that except for the special case of a constant ratio of attrition-rate coefficients,

---

*So-called after pioneering work of F. W. Lanchester [3].

the solutions to such differential equations cannot be represented in term of "elementary" functions of analysis. Thus, new transcendental functions arise in the study of combat modelled with time-dependent attrition-rate coefficients. In particular, we have previously introduced [5-6] so-called Lanchester-Clifford-Schläfli (LCS) functions for analyzing combat modelled with power attrition-rate coefficients with "no offset" (see Section 3 below).

In the Appendix to this report is contained a reduced set of tables for the LCS functions: it contains tables of five-decimal-place values of the hyperbolic-like LCS functions $F_{\alpha}(x)$, $H_{1-\alpha}(x)$, and $T_{\alpha}(x)$ (see Section 4 below) for 11 fractional values of α (see Section 6 below). A companion report [8] contains the most extensive set of tables currently available. The main body of this report provides the theoretical and modelling background for the use of these tables. In particular, we examine a model of a constant-speed attack on a static defensive position and show how associated range-dependent kill rates give rise to time-dependent attrition-rate coefficients with "no offset." Numerical computations are presented to illustrate the use of the LCS functions for analyzing such "aimed-fire" combat. As a consequence of the availability of these tables, one can now study this variable-coefficient combat model almost as easily and thoroughly as Lanchester's classic constant-coefficient model.

2. Variable-Coefficient Lanchester-Type Equations of Modern Warfare.

We consider combat between two homogeneous forces modelled by the following variable-coefficient Lanchester-type [3] (see [4,5]) equations of modern warfare

$$\begin{cases} \dfrac{dx}{dt} = -a(t)y & \text{with } x(0) = x_0 , \\ \dfrac{dy}{dt} = -b(t)x & \text{with } y(0) = y_0 , \end{cases} \tag{2.1}$$

where $t = 0$ denotes the time at which the battle begins, $x(t)$ and $y(t)$ denote the numbers of X and Y at time t, and $a(t)$ and $b(t)$ denote time-dependent Lanchester attrition-rate coefficients, which represent the effectiveness of each side's fire. These coefficients depend on variables such as force separation, tactical posture of targets, rate of target acquisition, firing rate, etc. (see [4-7] for further details). Variable attrition-rate coefficients are used to model temporal variations in firepower on the battlefield. In any analysis of combat, moreover, we should use the above equations (2.1) *only for* x *and* $y > 0$ and, *for example*, set $dx/dt = 0$ when $x = 0$, since negative force levels have no physical meaning.

Mathematically, we assume that the attrition-rate coefficients $a(t)$ and $b(t)$ are defined, positive, and continuous for $t_0 < t < +\infty$ with $t_0 \leq 0$. We also assume that $a(t)$ and $b(t) \in L(t_0,T)$ for any finite $T \geq t_0$. We further take $a(t)$ and $b(t)$ to be given in the form

$$a(t) = k_a g(t) , \qquad \text{and} \qquad b(t) = k_b h(t) , \tag{2.2}$$

where k_a and k_b are positive constants chosen so that $a(t)/b(t) = k_a/k_b$ when $g(t) \equiv h(t)$. We introduce the combat-intensity parameter λ_I and the relative-fire-effectiveness parameter λ_R defined by

$$\lambda_I = \sqrt{k_a k_b}\ , \qquad \text{and} \qquad \lambda_R = k_a/k_b\ . \tag{2.3}$$

From our assumptions about $a(t)$ and $b(t)$, it follows that, for example, $a(t) \notin L(t_0,T)$ implies $\int_{t_0}^{T} a(t)dt = +\infty$.

The X force level as a function of time may be represented as [5,6]

$$x(t) = x_0\{C_Y(0)C_X(t) - S_Y(0)S_X(t)\} - y_0\sqrt{\lambda_R}\ \{C_X(0)S_X(t) - S_X(0)C_X(t)\}, \tag{2.4}$$

where the hyperbolic-like <u>general Lanchester functions</u> (GLF) $C_X(t)$ and $S_X(t)$ are linearly-independent solutions to the X <u>force-level equation</u>

$$\frac{d^2x}{dt^2} - \left\{\frac{1}{a(t)}\frac{da}{dt}\right\}\frac{dx}{dt} - a(t)b(t)x = 0\ , \tag{2.5}$$

with initial conditions

$$C_X(t_0) = 1\ , \qquad S_X(t_0) = 0\ ,$$

$$\{1/a(t_0)\}\ dC_X/dt(t_0) = 0\ , \qquad \{1/a(t_0)\}\ dS_X/dt(t_0) = 1/\sqrt{\lambda_R}\ . \tag{2.6}$$

Here t_0 denotes the largest finite time at which $a(t)$ or $b(t)$ ceases to be defined, positive, or continuous. The Y force level as a function of time is given by a similar expression, with $C_Y(t)$ and $S_Y(t)$ being analogously defined for the corresponding Y force-level equation.

It is sometimes convenient to introduce the new independent variable τ defined by

$$\tau = \int_{t_0}^{t} \sqrt{a(s)b(s)}\, ds\ . \tag{2.7}$$

It is readily seen that the transformation $\tau = \tau(t)$ is well defined and invertible. Let us denote $\tau(0)$ as τ_0. We observe that $t_0 \leq 0$ implies that $\tau_0 \geq 0$. If we denote the "average intensity of combat" as $\overline{\sqrt{a(t)b(t)}}$, then

$$\overline{\sqrt{a(t)b(t)}}\ t = \{(1/t)\int_0^t \sqrt{a(s)b(s)}\, ds\}t = \tau - \tau_0\ . \tag{2.8}$$

The substitution (2.7) transforms (2.5) into

$$\frac{d^2x}{d\tau^2} - \left(\frac{1}{2}\right)\left\{\frac{d}{d\tau}\ \ell n\ R(\tau)\right\}\frac{dx}{d\tau} - x = 0\ , \tag{2.9}$$

with initial conditions

$$x(\tau_0) = x_0\ , \qquad \text{and} \qquad \{1/\sqrt{R(\tau_0)}\}\ dx/d\tau(\tau_0) = -y_0\ ,$$

where $R(\tau) = a(t)/b(t)$.

3. Combat Modelled with Power Attrition-Rate Coefficients.

The above equations (2.1) basically apply to "aimed-fire" combat when target-acquisition times do not depend on the numbers of targets available (see [5,6] for further details). A large class of tactical situations of interest can be modelled with the following general power attrition-rate coefficients [5-7]

$$a(t) = k_a(t + C)^{\mu}, \qquad \text{and} \qquad b(t) = k_b(t + C + A)^{\nu}, \tag{3.1}$$

where A and $C \geq 0$. We will call A the <u>offset parameter</u>, since it allows us to model (with μ and $\nu \geq 0$) battles between opposing weapon systems with different maximum effective ranges (see [5,6]). We will call C the <u>starting parameter</u>, since it allows us to model (again, with μ and $\nu \geq 0$) battles that begin within the maximum effective ranges of the two opposing systems. We observe that for the general power attrition-rate coefficients (3.1) we have $t_0 = -C$, and μ and ν must be > -1 in order that $a(t)$ and $b(t) \in L(t_0,T)$.

The above nomenclature is motivated and possible applications of our work are indicated by considering S. Bonder's model of the constant-speed attack on a static defensive position (see [4-7] for further details)

$$\frac{dx}{dt} = -\alpha(r)y, \qquad \text{and} \qquad \frac{dy}{dt} = -\beta(r)x , \tag{3.2}$$

where r denotes the range between opposing forces, and $\alpha(r)$ and $\beta(r)$ denote range-dependent attrition-rate coefficients. Range is related to time by

$$r(t) = R_0 - vt , \tag{3.3}$$

where R_0 denotes the opening range of battle and $v > 0$ denotes the constant attack speed. For example, let us consider the constant-speed attack of a homogeneous Y force against the static defensive position of a homogeneous X force. Figure 1 diagrammatically portrays this situation.

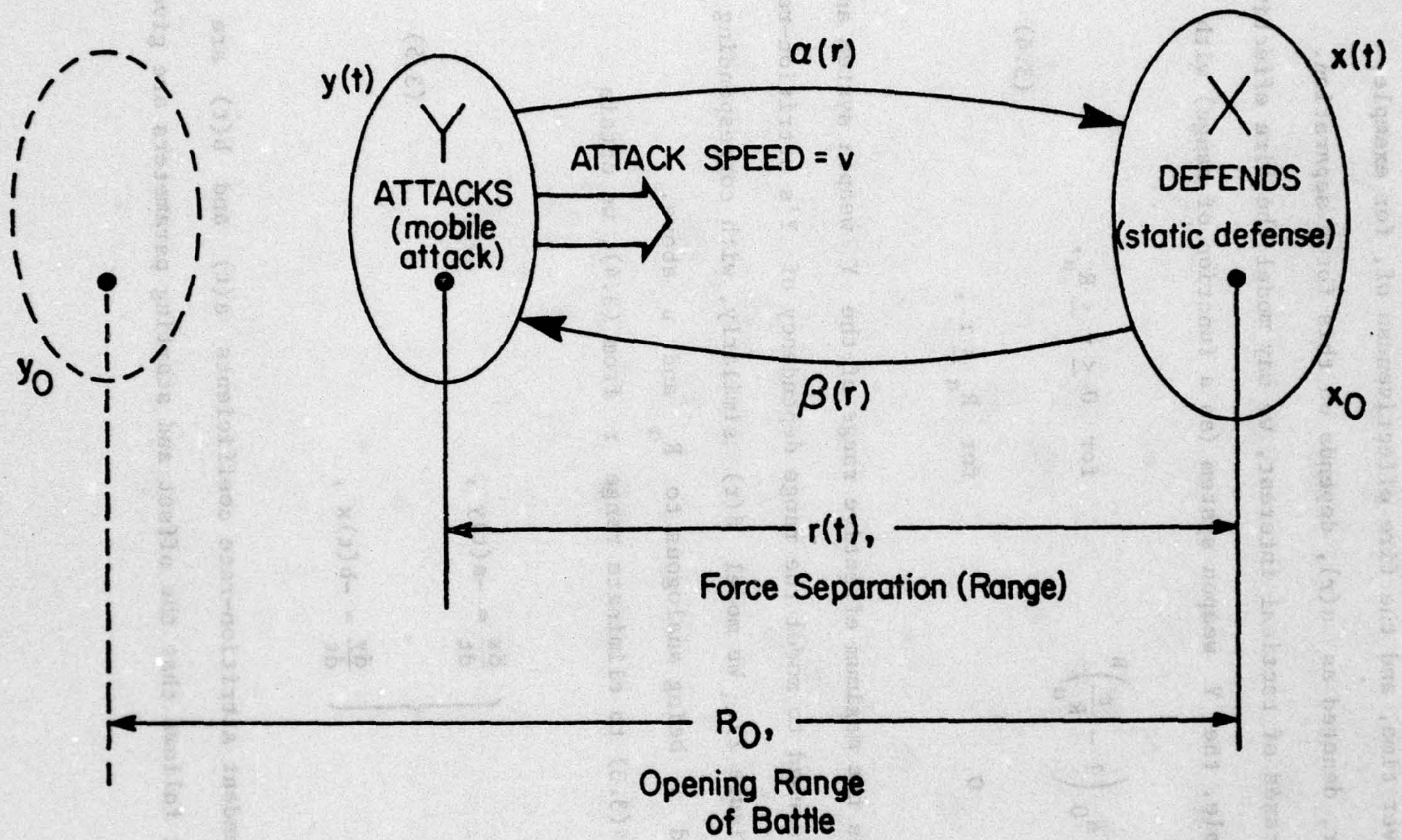

Figure 1. Diagram of Bonder's constant-speed attack model. Force separation, $r(t)$, is given by $r(t) = R_0 - vt$.

The basic idea is that force separation, i.e. range between the opposing forces, changes over time, and the fire effectiveness of, for example, a single Y firer, denoted as $\alpha(r)$, depends on this force separation.

In many cases of tactical interest, we may model the fire effectiveness of, for example, the Y weapon system (as a function of range) with

$$\alpha(r) = \begin{cases} \alpha_0 \left(1 - \dfrac{r}{R_\alpha}\right)^{\mu} & \text{for } 0 \leq r \leq R_\alpha, \\ 0 & \text{for } R_\alpha \leq r , \end{cases} \tag{3.4}$$

where R_α denotes the maximum effective range of the Y weapon system and $\mu \geq 0$. Here μ is used to model the range dependency of Y's attrition-rate coefficient (see Figure 2). We model $\beta(r)$ similarly, with corresponding quantities R_β and ν being analogous to R_α and μ above.

If we use (3.3) to eliminate range r from (3.4), we obtain

$$\begin{cases} \dfrac{dx}{dt} = -a(t)y , \\ \dfrac{dy}{dt} = -b(t)x , \end{cases} \tag{3.5}$$

where the time-dependent attrition-rate coefficients $a(t)$ and $b(t)$ are given by (3.1). It follows that the offset and starting parameters are given by

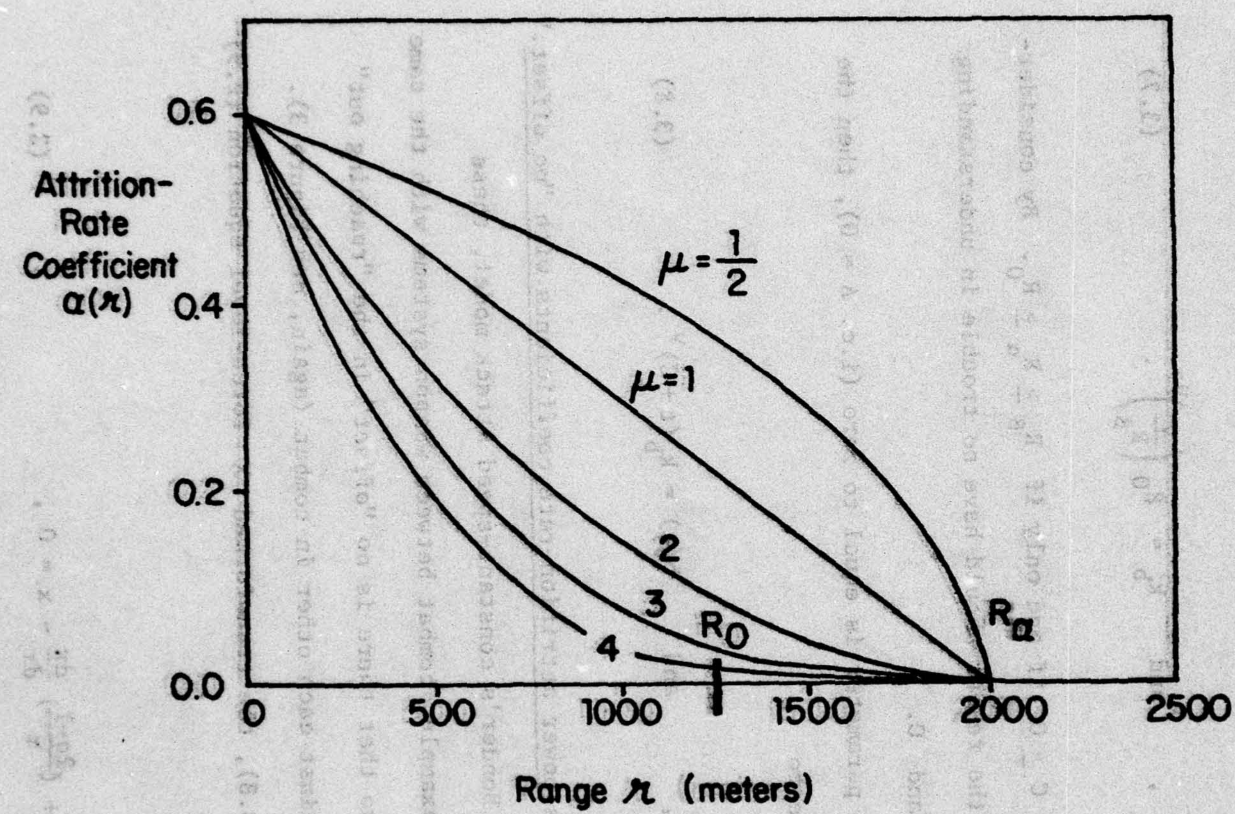

Figure 2. Dependence of Y's attrition-rate coefficient $\alpha(r)$ on the exponent μ with the maximum effective range of the weapon system and kill rate at zero range held constant. [NOTES: 1. The maximum effective range of the system is denoted as $R_\alpha = 2000$ meters. 2. $\alpha(0) = \alpha_0 = 0.6X$ casualties/(unit time × number of Y firers) denotes the weapon-system kill rate for Y at zero force separation (range). 3. The opening range of battle is denoted as $R_0 = 1250$ meters and (as shown) $R_0 < R_\alpha$.]

$$A = \left(\frac{R_\beta - R_\alpha}{v}\right), \quad \text{and} \quad C = \left(\frac{R_\alpha - R_0}{v}\right), \tag{3.6}$$

and that

$$k_a = \alpha_0 \left(\frac{v}{R_\alpha}\right)^\mu , \quad \text{and} \quad k_b = \beta_0 \left(\frac{v}{R_\beta}\right)^\nu . \tag{3.7}$$

We observe that A and $C \geq 0$ if and only if $R_\beta \geq R_\alpha \geq R_0$. By considering (3.6) and Figure 3, the reader should have no trouble in understanding our terminology for A and C.

When the offset parameter is equal to zero (i.c. $A = 0$), then the coefficients (3.1) reduce to

$$a(t) = k_a(t+C)^\mu, \quad \text{and} \quad b(t) = k_b(t+C)^\nu . \tag{3.8}$$

We will refer to (3.8) as power attrition-rate coefficients with "no offset." As we have seen above in Bonder's constant-speed attack model, these coefficients model, for example, combat between weapon systems with the same maximum effective range so that there is no "offset" in the "reaching out" of the weapon systems against each other in combat (again, see Figure 3). For these coefficients (3.8), the transformed X force-level equation (2.9) becomes

$$\frac{d^2x}{d\tau^2} + \left(\frac{2q-1}{\tau}\right) \frac{dx}{d\tau} - x = 0 , \tag{3.9}$$

with initial conditions

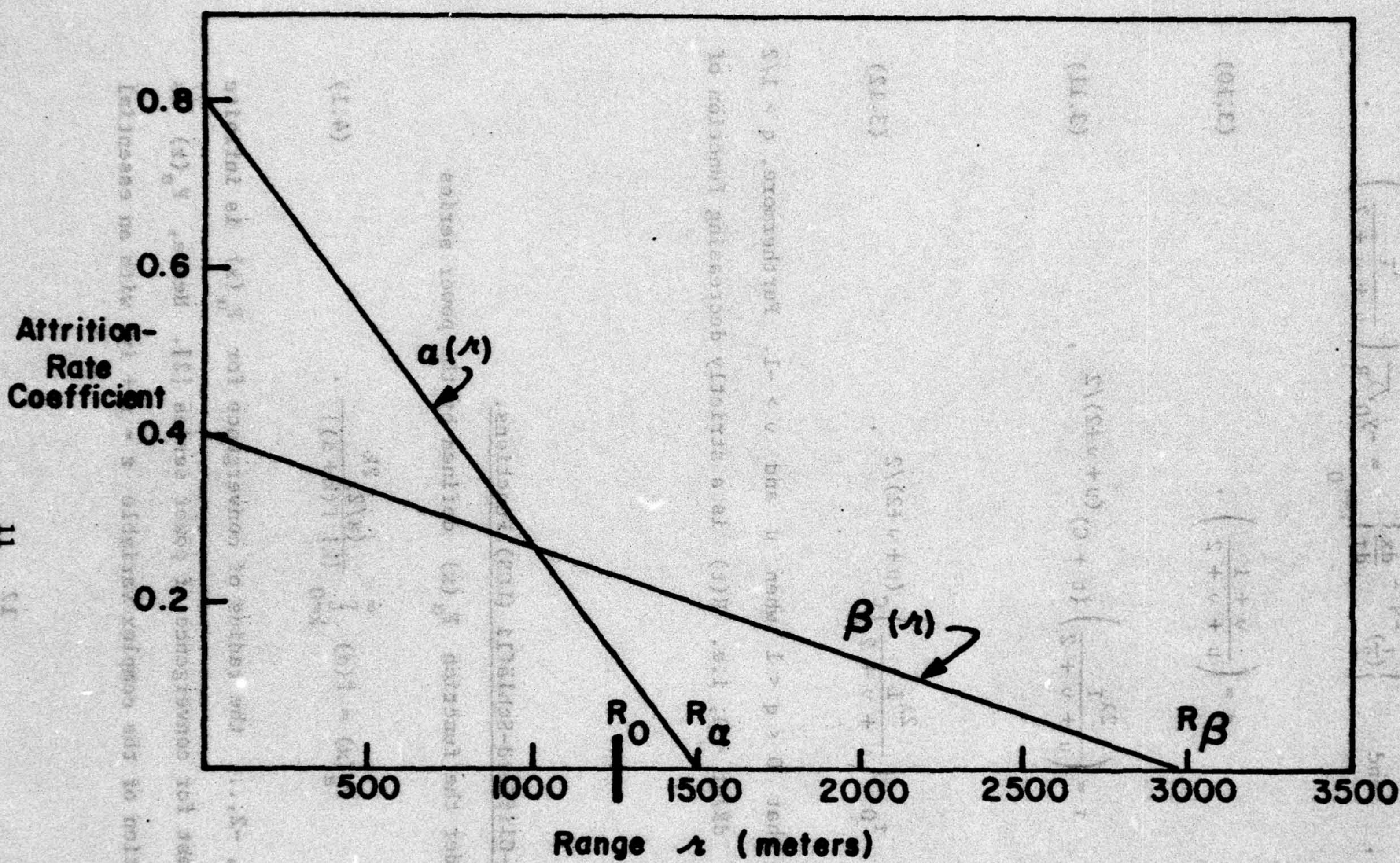

Figure 3. Explanation of the offset parameter A and the starting parameter C for power attrition-rate coefficients modelling constant-speed attack. [NOTES: 1. The maximum effective ranges of the X and Y weapon systems are denoted as R_α and R_β, respectively. 2. The opening range of battle is denoted as R_0 and (as shown) $R_0 < \text{minimum}(R_\alpha, R_\beta)$. 3. The offset parameter is given by $A = (R_\beta - R_\alpha)/v$. 4. The starting parameter is given by $C = (R_\alpha - R_0)/v$.]

$$x(\tau_0) = x_0 \,, \quad \text{and} \quad \left\{ \left(\frac{\tau}{2}\right)^{2q-1} \frac{dx}{d\tau} \right\}_{\tau=\tau_0} = -y_0\sqrt{\lambda_R} \left(\frac{\lambda_I}{\mu + \nu + 2} \right)^{2q-1} .$$

Here

$$q = \left(\frac{\nu + 1}{\mu + \nu + 2} \right) , \tag{3.10}$$

$$\tau = \left(\frac{2\lambda_I}{\mu + \nu + 2} \right) (t + C)^{(\mu+\nu+2)/2} , \tag{3.11}$$

and

$$\tau_0 = \left(\frac{2\lambda_I}{\mu + \nu + 2} \right) C^{(\mu+\nu+2)/2} . \tag{3.12}$$

Let us observe that $0 < q < 1$ when μ and $\nu > -1$. Furthermore, $q > 1/2$ if and only if $dR/dt < 0$, i.e. $R(t)$ is a strictly decreasing function of time.

4. Lanchester-Clifford-Schläfli (LCS) Functions.

Consider the function $F_\alpha(x)$ defined by the power series

$$F_\alpha(x) = \Gamma(\alpha) \sum_{k=0}^{\infty} \frac{(x/2)^{2k}}{\{k!\ \Gamma(k+\alpha)\}} . \tag{4.1}$$

For $\alpha \neq 0, -1, -2, \ldots$ the radius of convergence for $F_\alpha(x)$ is infinite by the ratio test for convergence of power series [2]. Hence, $F_\alpha(z)$ is an entire function of the complex variable $z = x + iy$, with an essential

singularity at the point at infinity. Now consider the function $H_\alpha(x)$ defined by the infinite series

$$H_\alpha(x) = \Gamma(\alpha) \sum_{k=0}^{\infty} \frac{(x/2)^{2(k+\alpha)}}{\{k!\ \Gamma(k+\alpha+1)\}} . \tag{4.2}$$

Observing that

$$H_\alpha(x) = (1/\alpha)(x/2)^{2\alpha}\ F_{\alpha+1}(x) , \tag{4.3}$$

we see that for $\alpha > 0$ the infinite series (4.2) is uniformly convergent on compact subsets of the complex plane. From (4.3) one can readily deduce the recursive relation

$$F_\alpha(x) = F_{\alpha+1}(x) + \left\{\frac{(x/2)^2}{\alpha(\alpha+1)}\right\} F_{\alpha+2}(x) . \tag{4.4}$$

We will call the functions $F_\alpha(x)$ and $H_\alpha(x)$ <u>Lanchester-Clifford-Schläfli</u> (LCS) functions (see Note 10 on pp. 66-67 of [5]). Other properties are readily deduced and are given in Table I.

The function $F_\alpha(x)$ satisfies the linear second-order ordinary differential equation

$$\frac{d^2F_\alpha}{dx^2} + \left(\frac{2\alpha-1}{x}\right)\frac{dF_\alpha}{dx} - F_\alpha = 0 , \tag{4.5}$$

with initial conditions

Table I. Properties of the LCS Functions $F_\alpha(x)$ and $H_\alpha(x)$.

1. $dF_\alpha/dx = (x/2)^{1-2\alpha} H_\alpha(x)$

2. $dH_\alpha/dx = (x/2)^{2\alpha-1} F_\alpha(x)$

3. $F_\alpha(x) F_{1-\alpha}(x) - H_\alpha(x) H_{1-\alpha}(x) = 1 \quad \forall x$

   where α is not an integer (including zero)

4. $F_\alpha(x=0) = 1$

5. $H_\alpha(x=0) = 0 \quad$ for $\alpha > 0$

6. $dF_\alpha/dx(x=0) = 0$

7. $\{(x/2)^{1-2\alpha} dH_\alpha/dx\}_{x=0} = 1$

8. $F_{1/2}(x) = \cosh x$

9. $H_{1/2}(x) = \sinh x$

$$F_\alpha(0) = 1, \qquad \text{and} \qquad \frac{dF_\alpha}{dx}(0) = 0 ,$$

while $H_\alpha(x)$ satisfies

$$\frac{d^2H_\alpha}{dx^2} - \left(\frac{2\alpha-1}{x}\right)\frac{dH_\alpha}{dx} - H_\alpha = 0 , \tag{4.6}$$

with initial conditions

$$H_\alpha(0) = 0 , \qquad \text{and} \qquad \left\{\left(\frac{x}{2}\right)^{1-2\alpha}\frac{dH_\alpha}{dx}\right\}_{x=0} = 1 .$$

Thus, $\{F_\alpha, H_{1-\alpha}\}$ is a fundamental system of solutions to

$$\frac{d^2F}{dx^2} + \left(\frac{2\alpha-1}{x}\right)\frac{dF}{dx} - F = 0 , \tag{4.7}$$

with Wronskian $W(F_\alpha, H_{1-\alpha}) = (x/2)^{1-2\alpha}$. It follows that the GLF for the X and Y force-level equations for combat modelled with the attrition-rate coefficients (3.8) are given by

$$C_X(t) = F_q(\tau(t)), \qquad S_X(t) = \left(\frac{\lambda_I}{\mu+\nu+2}\right)^{2q-1} H_p(\tau(t)) , \tag{4.8}$$

$$C_Y(t) = F_p(\tau(t)), \qquad S_Y(t) = \left(\frac{\lambda_I}{\mu+\nu+2}\right)^{1-2q} H_q(\tau(t)) , \tag{4.9}$$

where $p = 1-q$. If we define

$$T_\alpha(x) = H_{1-\alpha}(x)/F_\alpha(x) \,, \tag{4.10}$$

then

$$T_X(t) = \frac{S_X(t)}{C_X(t)} = \left(\frac{\lambda_I}{\mu + \nu + 2}\right)^{2q-1} \frac{H_p(\tau(t))}{F_q(\tau(t))} \,, \tag{4.11}$$

or

$$T_X(t) = \left(\frac{\lambda_I}{\mu + \nu + 2}\right)^{2q-1} T_q(\tau(t)) \,, \tag{4.12}$$

where $T_X(t)$ denotes a hyperbolic-like GLF, which corresponds to the hyperbolic tangent. Observing that for $\mu, \nu > -1$, $\lim_{\tau \to +\infty} \tau(t) = +\infty$, we see that $T_\alpha(x)$ is a strictly increasing function of x on the interval $[0, +\infty)$ and

$$0 \leq T_\alpha(x) < \frac{\Gamma(1-\alpha)}{\Gamma(\alpha)} \qquad \text{for } 0 \leq x < +\infty \,, \tag{4.13}$$

with

$$\lim_{x \to +\infty} T_\alpha(x) = \frac{\Gamma(1-\alpha)}{\Gamma(\alpha)} \,, \tag{4.14}$$

since by the results of Taylor and Comstock [7] the parity-condition parameter $Q^* = Q^*(\mu, \nu, C = 0)$ is given by

$$\lim_{t \to +\infty} T_X(t) = \frac{1}{Q^*(\mu, \nu, 0)} = \left(\frac{\lambda_I}{\mu + \nu + 2}\right)^{2q-1} \frac{\Gamma(p)}{\Gamma(q)} \,. \tag{4.15}$$

We recall that Taylor and Comstock [7] have introduced the so-called parity-condition parameter Q^* as the value (or range of such values) for the initial condition Q to the initial-value problem

$$\begin{cases} \dfrac{dE_X^-}{dt} = -\dfrac{1}{\sqrt{\lambda_R}}\, a(t) E_Y^- & \text{with } E_X^-(t_0) = 1\,, \\ & \qquad\qquad (4.16) \\ \dfrac{dE_Y^-}{dt} = -\sqrt{\lambda_R}\, b(t) E_X^- & \text{with } E_Y^-(t_0) = Q\,, \end{cases}$$

such that $E_X^-(t;Q^*)$ and $E_Y^-(t;Q^*) > 0$ for all $t \geq t_0$. In other words, Q^* is the value of Q in (4.16) above such that neither E_X^- nor E_Y^- ever become zero. In this case, both $E_X^-(t;Q^*)$ and $E_Y^-(t;Q^*)$ are positive, strictly decreasing functions, similar to decreasing exponentials. Thus, we may call Q^* "the Y equivalent of an X force of unit strength," since the forces are "at parity," with neither force being annihilated in finite time. Taylor and Comstock have shown that for either $a(t) \notin L(0,+\infty)$ or $b(t) \notin L(0,+\infty)$, then Q^* is unique and given by

$$\lim_{t \to +\infty} \frac{S_X(t)}{C_X(t)} = \frac{1}{Q^*} \quad . \qquad (4.17)$$

The significance of the parity-condition parameter Q^* is that it allows us to predict force annihilation as the following theorem shows.

THEOREM 1 (Taylor and Comstock [7]): Assume that either $a(t) \notin L(0,+\infty)$ or $b(t) \notin L(0,+\infty)$. Then the X force will be annihilated in finite time if and only if

$$\frac{x_0}{y_0} < \sqrt{\lambda_R} \left\{ \frac{C_X(0) - Q^* S_X(0)}{Q^* C_Y(0) - S_Y(0)} \right\} \quad . \qquad (4.18)$$

5. Use of LCS Functions for Analyzing Combat.

The Lanchester-Clifford-Schläfli (LCS) functions $F_\alpha(x)$ and $H_\alpha(x)$ are useful for analyzing "aimed-fire" combat (see Section 3 above) modelled with the power attrition-rate coefficients with "no offset" (3.8), which we rewrite here as

$$a(t) = k_a(t + C)^\mu, \qquad \text{and} \qquad b(t) = k_b(t + C)^\nu . \tag{5.1}$$

In other words, the LCS functions arise in solving the differential combat model (2.1) with attrition-rate coefficients (5.1). In order that both $a(t)$ and $b(t) \in L(t_0,T)$, we must have μ and $\nu > -1$. Military situations modelled by these equations have been discussed in Section 3 above, e.g. combat between two weapon systems with the same maximum effective range. *For such combat, the LCS functions may be used to*

(1) compute force-level declines,

(2) predict force annihilation,

and (3) predict the time of force annihilation.

Let us now see how the LCS functions may be used to obtain the above information about force-level declines and force-annihilation prediction. According to (2.4), (4.8), and (4.9) above, the X force level is given by

$$x(t) = x_0\{F_p(\tau_0)\, F_q(\tau(t)) - H_q(\tau_0)\, H_p(\tau(t))\}$$

$$- y_0 \sqrt{\lambda_R} \left(\frac{\lambda_I}{\mu + \nu + 2}\right)^{2q-1} \{F_q(\tau_0)H_p(\tau(t)) - H_p(\tau_0)F_q(\tau(t))\}, \tag{5.2}$$

where q is given by (3.10), $p = 1-q$, and $\tau(t)$ is given by (3.11), which we rewrite as

$$\tau(t) = \left(\frac{2\lambda_I}{\mu + \nu + 2}\right)(t + C)^{(\mu+\nu+2)/2} . \tag{5.3}$$

The time to annihilate the X force* is determined by $x(t_a^X) = 0$, and it follows that

$$T_q(\tau(t_a^X)) = \frac{x_0 F_p(\tau_0) + y_0\sqrt{\lambda_R}\left(\frac{\lambda_I}{\mu+\nu+2}\right)^{q-p} H_p(\tau_0)}{x_0 H_q(\tau_0) + y_0\sqrt{\lambda_R}\left(\frac{\lambda_I}{\mu+\nu+2}\right)^{q-p} F_q(\tau_0)} , \tag{5.4}$$

where from (4.10)

$$T_q(\tau(t)) = H_p(\tau(t))/F_q(\tau(t)) , \tag{5.5}$$

and we recall that $p + q = 1$. It follows that the time to annihilate X, t_a^X, is given by

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*If we multiply the first equation of (2.1) by y, the second by x, add, and integrate, we obtain

$$x(t)\, y(t) = x_0 y_0 - \int_0^t \{a(s)\, y^2(s) + b(s)\, x^2(s)\}ds ,$$

which shows that $x(t)$ and $y(t)$ can have at most one finite zero. Hence, if $x(t_a^X) = 0$, then we know that $y(t) > 0$ for all $t \geq 0$.

$$t_a^X = \tau^{-1}\left\{T_q^{-1}\left[\frac{x_0 F_p(\tau_0) + y_0\sqrt{\lambda_R}\left(\frac{\lambda_I}{\mu+\nu+2}\right)^{q-p} H_p(\tau_0)}{x_0 H_q(\tau_0) + y_0\sqrt{\lambda_R}\left(\frac{\lambda_I}{\mu+\nu+2}\right)^{q-p} F_q(\tau_0)}\right]\right\}. \quad (5.6)$$

Taylor and Comstock [7] have shown that $T_q(\tau)$ is strictly increasing and satisfies (see also (4.12) above)

$$0 \leq T_q(\tau) < \Gamma(p)/\Gamma(q), \quad (5.7)$$

where $p = 1-q$. It follows that in order for X to be annihilated in finite time, the right-hand side of (5.4) must be less than $\Gamma(p)/\Gamma(q)$. Let us observe that for $t_0 = -C = 0$, (5.4) simplifies to

$$T_q(\tau(t_a^X)) = \frac{x_0}{y_0\sqrt{\lambda_R}}\left(\frac{\lambda_I}{\mu+\nu+2}\right)^{p-q}. \quad (5.8)$$

Thus, we have proved the following theorem concerning force-annihilation prediction.

THEOREM 2: Consider combat between two homogeneous forces modelled by (2.1) with power attrition-rate coefficients (5.1). Assume that μ and $\nu > -1$ and that the above equations hold for all time. Then the X force will be annihilated in finite time if and only if

$$\Gamma(q)\left\{x_0F_p(\tau_0) + y_0\sqrt{\lambda_R}\left(\frac{\lambda_I}{\mu+\nu+2}\right)^{q-p} H_p(\tau_0)\right\}$$

$$< \Gamma(p)\left\{x_0H_q(\tau_0) + y_0\sqrt{\lambda_R}\left(\frac{\lambda_I}{\mu+\nu+2}\right)^{q-p} F_q(\tau_0)\right\}, \qquad (5.9)$$

where $q = (\nu + 1)/(\mu + \nu + 2)$ and $p = 1-q$. For $t_0 = 0$ (i.e. $C = 0$ so that $\tau_0 = 0$), X will be annihilated in finite time if and only if

$$\frac{x_0}{y_0} < \frac{\Gamma(p)}{\Gamma(q)}\sqrt{\lambda_R}\left(\frac{\lambda_I}{\mu+\nu+2}\right)^{q-p} . \qquad (5.10)$$

6. Tabulation of LCS Functions.

This report contains a reduced set of tables of the Lanchester-Clifford-Schläfli functions. The Appendix contains tables of five-decimal-place values of the hyperbolic-like LCS functions $F_\alpha(x)$, $H_{1-\alpha}(x)$, and $T_\alpha(x)$ for various values of the argument x, namely x = 0.00 (0.01) 2.00 (0.1) 10.0, and α = 1/2, 1/3, 2/3, 1/4, 3/4, 1/5, 2/5, 3/5, 4/5, 3/7, and 4/7. These values of the index α correspond to μ, ν = 0, 1, 2, and 3 in (3.8) and allow one to analyze, for example, a basic spectrum of range capabilities for weapon systems in the constant-speed-attack model of Section 3. These tables have been calculated by the recursive means given in Section 8 of [5]. A more extensive tabulation (namely, for α = 1/2, 1/3, 2/3, 1/4, 3/4, 1/5, 2/5, 3/5, 4/5, 2/7, 3/7, 4/7, 5/7, 4/9, 5/9, 3/11, 5/11, 6/11, 8/11, 5/13, 8/13, 5/17, 12/17, 5/21, and 16/21 corresponding to μ, ν = 0, 1/4, 1/2, 1, $1\frac{1}{2}$, 2, 3)

is to be found in a companion report [8]. This companion report contains the most extensive set of tables of the Lanchester-Clifford-Schläfli functions currently available.

A representative tabulation of the hyperbolic-like LCS functions $F_\alpha(x)$, $H_{1-\alpha}(x)$, and $T_\alpha(x)$ is given in, for example, Tables 8A and 8B of the Appendix for $\alpha = 3/5$. The values of the argument x are the same as those used for the tabulation of the hyperbolic functions by Abramowitz and Stegun [1]. We observe from Table 8B and (4.13) that the limiting value of $T_\alpha(x)$ as $x \to +\infty$ (here $\alpha = 3/5$) is quickly reached, with three-decimal-place accuracy already attained for $x = 4.5$. Moreover, the use of these tables (specifically, Tables 8A and 8B of the Appendix) for combat analysis is illustrated in the next section.

7. Numerical Examples

In this section we examine a couple of numerical examples to show some of the insights that may be gained into the dynamics of combat between two homogeneous forces from our results (see also [6]). These examples illustrate the use of the LCS functions $F_\alpha(x)$, $H_{1-\alpha}(x)$, and $T_\alpha(x)$ for analyzing "aimed-fire" combat modelled with the power attrition-rate coefficients with "no offset" (5.1). As in [4-7], we consider S. Bonder's model (3.2) for the constant-speed attack against a static defensive position. We will focus on the use of the LCS functions for predicting force annihilation, since the computing of force-level trajectories with Lanchester functions is adequately handled elsewhere (see [4-5]).

Let us accordingly consider the constant-speed attack of a homogeneous Y force against the static defensive position of a homogeneous X force (see Section 3 above for further modelling details, especially Figure 1). For our numerical computations, we assume that the fire effectiveness of the Y weapon system varies linearly with range, i.e.

$$\alpha(r) = \begin{cases} \alpha_0\left(1 - \dfrac{r}{R_\alpha}\right) & \text{for } 0 \le r \le R_\alpha, \\ 0 & \text{for } R_\alpha \le r, \end{cases} \tag{7.1}$$

and that the fire effectiveness of the X weapon system varies quadratically with range, i.e.

$$\beta(r) = \begin{cases} \beta_0\left(1 - \dfrac{r}{R_\beta}\right)^2 & \text{for } 0 \le r \le R_\beta, \\ 0 & \text{for } R_\beta \le r, \end{cases} \tag{7.2}$$

with $R_\alpha = R_\beta$, i.e. both weapon systems have the same maximum effective range. In other words, $\mu = 1$ in (3.4) and $\nu = 2$ for $\beta(r)$. We consider a battle modelled by the input data given in Table II. In terms of time as the independent variable, the attrition-rate coefficients (7.1) and (7.2) become via (3.3)

$$a(t) = k_a(t + C) \qquad \text{and} \qquad b(t) = k_b(t + C)^2, \tag{7.3}$$

Table II. Input Data for Numerical Examples

$\mu = 1, \quad \nu = 2$

$\alpha_0 = 0.06$ X casualties/minute/Y firer

$\beta_0 = 0.6$ Y casualties/minute/X firer

$R_\alpha = R_\beta = 2000$ meters

$v = 5$ miles/hour

where $R_\alpha = R_\beta$,

$$C = \frac{R_\alpha - R_0}{v}, \quad k_a = \frac{\alpha_0 v}{R_\alpha}, \quad \text{and} \quad k_b = \beta_0 \left(\frac{v}{R_\beta}\right)^2 . \tag{7.4}$$

From the input data given in Table II, we compute the parameter values shown in Table III, since the transformed X force-level equation is given by (3.9) with $q = (\nu + 1)/(\mu + \nu + 2)$, $p = 1-q$, $\mu = 1$, and $\nu = 2$. Thus, the X force level may be computed with $F_\alpha(\tau)$ and $H_{1-\alpha}(\tau)$ with $\alpha = q = 3/5$. Force-annihilation prediction involves the limiting value of $T_\alpha(\tau) = H_{1-\alpha}(\tau)/F_\alpha(\tau)$ as $\tau \to +\infty$. From Table 8B of the Appendix and Table III, we note the predicted agreement between $\Gamma(1-\alpha)/\Gamma(\alpha)$ and the limiting value of $T_\alpha(x)$ as $x \to +\infty$ [recall (4.13)] for $\alpha = q = 3/5$. We now consider two cases: (I) $R_0 = 2000$ meters, and (II) $R_0 = 1250$ meters.

<u>When $R_0 = 2000$ meters</u> (see Figure 3 of [4]), we have $C = 0$ and $\tau_0 = 0$. The maximum time that the battle can last is $t_{max} = R_0/v = 14.91$ minutes, since at this time the attackers reach their final objective, i.e. the defender's position (again, see Figure 1). We now consider the qualitative behavior of the $\mu = 1$, $\nu = 2$ force-level trajectory shown in Figure 3 of [4]. Theorem 2 tells us that the X force can be annihilated in finite time if and only if

$$\frac{x_0}{y_0} < \frac{\Gamma(p)}{\Gamma(q)} \sqrt{\lambda_R} \left(\frac{\lambda_I}{\mu + \nu + 2}\right)^{q-p} , \tag{7.3}$$

where $q = 3/5$ and $p = 1-q$. Using the numerical values in Table III, we compute from (7.3) that the X force can be annihilated in finite time if and only if

Table III. Parameter Values for Numerical Examples

$k_a = 4.0233 \times 10^{-3}$ X casualties/(minute)$^{\mu}$/Y firer

$k_b = 2.6979 \times 10^{-3}$ Y casualties/(minute)$^{\nu}$/X firer

$p = 2/5, \quad q = 3/5$

$\Gamma(p)/\Gamma(q) = 1.48951$

$A = 0$

$$\frac{x_0}{y_0} < 0.420 \ . \tag{7.4}$$

When the X force can be annihilated, its annihilation time is given by (5.8), which we rewrite here as

$$T_q(\tau(t_a^X)) = \frac{x_0}{y_0 \sqrt{\lambda_R}} \left(\frac{\lambda_I}{\mu + \nu + 2} \right)^{p-q} , \tag{7.5}$$

where

$$\tau(t) = \left(\frac{2\lambda_I}{\mu + \nu + 2} \right) t^{(\mu+\nu+2)/2} \ . \tag{7.6}$$

Thus, for the numerical values given in Table III, the time of annihilation of the X force is given by

$$T_q(\tau(t_a^X)) = 3.544 \frac{x_0}{y_0} , \tag{7.7}$$

with $q = 3/5$. We will now illustrate further computations for $x_0 = 10$ and $y_0 = 30$. From (7.4) we see that the X force can be annihilated in finite time (but we must verify that $t_a^X \leq t_{max}$). In this case (7.7) becomes

$$T_q(\tau(t_a^X)) = 1.18122 \ . \tag{7.8}$$

We must now determine $\tau(t_a^X)$ such that $\tau(t_a^X) = T_q^{-1}(1.18122)$ by using interpolation methods and Tables 8A and 8B. From Table 8A, we find

$$T_q(\tau) = 1.18172 \qquad \text{for} \quad = 1.01$$

$$T_q(\tau) = 1.17630 \qquad \text{for} \quad = 1.00$$

so that using linear interpolation, we obtain

$$\tau(t_a^X) = 1.009 \ , \tag{7.9}$$

whence use of (7.6) yields

$$t_a^X = 14.24 \text{ minutes}, \tag{7.10}$$

which is less than $t_{max} = 14.91$ minutes so that the defending X force is indeed annihilated before the attacking Y force reaches its final objective. Since $r(t) = R_0 - vt$, we find that force separation at the instant of annihilation of the X force is

$$r_a^X = 89.8 \text{ meters} \ . \tag{7.11}$$

Further results may be computed in a similar fashion and are given in Table IV.

When $R_0 = 1250$ meters (see Figure 3 of [5]), we have $C = 5.5923$ minutes, $\tau_0 = 0.0975$, and $t_{max} = R_0/v = 9.32$ minutes. In this case Theorem 2 tells us that the X force can be annihilated in finite time if and only if

Table IV. Annihilation of the X Force as a Function of the Initial Force Ratio for R_0 = 2000 meters

(x_0/y_0)	t_a^X (minutes)	r_a^X (meters)
0.333	14.24	89.8
0.250	11.61	443.2
0.200	10.19	633.2

$$\frac{x_0}{y_0} < \sqrt{\lambda_R}\left(\frac{\lambda_I}{\mu + \nu + 2}\right)^{q-p} \frac{\Gamma(p)}{\Gamma(q)} \frac{\left\{F_q(\tau_0) - \frac{\Gamma(q)}{\Gamma(p)} H_p(\tau_0)\right\}}{\left\{F_p(\tau_0) - \frac{\Gamma(p)}{\Gamma(q)} H_q(\tau_0)\right\}} , \tag{7.12}$$

with $q = 3/5$ and $p = 1-q$. Using linear interpolation, we obtain from Tables 7A and 8A of the Appendix that for the numerical values of Table III

$$\begin{aligned} F_p(\tau_0) &= 1.006 , & H_q(\tau_0) &= 0.044 , \\ F_q(\tau_0) &= 1.004 , & H_p(\tau_0) &= 0.223 , \end{aligned} \tag{7.13}$$

so that (7.12) says that the X force can be annihilated if and only if

$$\frac{x_0}{y_0} < 0.382 . \tag{7.14}$$

When the X force can be annihilated, its annihilation time is given by (5.4), which we rewrite here as

$$T_q(\tau(t_a^X)) = \frac{\left\{\frac{x_0}{y_0\sqrt{\lambda_R}}\left(\frac{\lambda_I}{\mu + \nu + 2}\right)^{p-q} F_p(\tau_0) + H_p(\tau_0)\right\}}{\left\{F_q(\tau_0) + \frac{x_0}{y_0\sqrt{\lambda_R}}\left(\frac{\lambda_I}{\mu + \nu + 2}\right)^{p-q} H_q(\tau_0)\right\}} , \tag{7.15}$$

whence for the data of Table III

$$T_a(\tau(t_a^X)) = \frac{3.565u_0 + 0.223}{0.156u_0 + 1.004} , \tag{7.16}$$

where $u_0 = x_0/y_0$. Let us also record here that (3.11) yields

$$t = \left(\frac{\{\mu + \nu + 2\}\tau}{2\lambda_I}\right)^{2/(\mu+\nu+2)} - C \,. \tag{7.17}$$

We will again illustrate further computations for $x_0 = 10$ and $y_0 = 30$. From (7.14) we see that the X force can be annihilated in finite time (but again we must investigate whether or not $t_a^X \leq t_{max}$). In this case (7.16) becomes

$$T_q(\tau(t_a^X)) = 1.33651 \,, \tag{7.18}$$

whence Table 8A of the Appendix and linear interpolation yield

$$\tau(t_a^X) = 1.397 \,, \tag{7.19}$$

so that by (7.17)

$$t_a^X = 10.63 \text{ minutes} \,. \tag{7.20}$$

Since $t_{max} = R_0/v = 9.32$ minutes $< t_a^X$, we see that the attacking Y force overruns the defender's position before annihilation of the X force occurs. Thus, the battle ends with $x_f = x(t_f) > 0$ and $y_f > 0$ at $t_f = t_{max} = 9.32$ minutes. Corresponding to $t_f = 9.32$ minutes is $\tau_f = 1.1318$, and then Table 8A of the Appendix yields

$$F_q(\tau_f = 1.1318) = 1.589 \,, \qquad H_p(1.1318) = 1.973 \,, \tag{7.21}$$

whence via (2.4), (4.8), (4.9), and (7.13) we obtain

$$x_f = x(t_f) = x(r = 0) = 1.35 \,. \tag{7.22}$$

Some further numerical results are given in Table V. Again, these parametric results should be contrasted with the single $\mu = 1$, $\nu \neq 2$ force-level trajectory shown in Figure 3 of [5].

8. Final Remarks

In the previous section above, we have seen how the LCS functions allow one to conveniently obtain much valuable information about the model (2.1) with power attrition-rate coefficients (3.8) without having to explicitly compute the entire force-level trajectories. Previously we were limited to computing only force-level trajectories (see [4-5]). With the availability of these tabulations of LCS functions (see the Appendix of this report and [8]), we can now tell who is going to be annihilated and when this event will happen without having to compute the trajectories. Not only did we answer questions about the qualitative behavior of the model (e.g. force annihilation) for specific values of, for example, initial force levels but also for a range of values of the initial force ratio (i.e. parametric analysis of model behavior).

Table V. Annihilation of the X Force as a Function of the Initial Force Ratio for R_0 = 1250 meters

(x_0/y_0)	t_a^X (minutes)	r_a^X (meters)
0.333	10.63	______†
0.250	7.56	235.9
0.200	6.17	422.8

† t_{max} = 9.32 minutes and $x_f = x(r=0) = 1.35$.

The results of this report may be used for other parametric analyses, e.g. parametric dependence of battle outcome on attrition-rate coefficients. Thus, the contents of this report allow one to develop important insights into the dynamics of combat between two homogeneous forces with temporal variations in fire effectiveness. With the availability of tabulations of the LCS functions, one can now analyze such combat modelled by the power attrition-rate coefficients (3.8) with somewhat the same facility as he can for the constant-coefficient case and thus aid in parametric analyses. For further discussions of the significance of such results for military operations research, the reader is directed to [6] and [7].

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APPENDIX: Tabulation of the LCS Functions $F_\alpha(x)$, $H_{1-\alpha}(x)$, and $T_\alpha(x)$ for α = 1/2, 1/3, 2/3, 1/4, 3/4, 1/5, 2/5, 3/5, 4/5, 3/7, and 4/7.

$\alpha = 1/2$

x	$F_{1/2}(x)$	$H_{1/2}(x)$	$T_{1/2}(x)$	x	$F_{1/2}(x)$	$H_{1/2}(x)$	$T_{1/2}(x)$	x	$F_{1/2}(x)$	$H_{1/2}(x)$	$T_{1/2}(x)$
0.0	1.00000	0.0	0.0	0.50	1.12763	0.52110	0.46212	1.00	1.54308	1.17520	0.76159
0.01	1.00005	0.01000	0.01000	0.51	1.13289	0.53240	0.46995	1.01	1.55491	1.19069	0.76576
0.02	1.00020	0.02000	0.02000	0.52	1.13827	0.54375	0.47770	1.02	1.56689	1.20630	0.76987
0.03	1.00045	0.03000	0.02999	0.53	1.14377	0.55516	0.48538	1.03	1.57904	1.22203	0.77391
0.04	1.00080	0.04001	0.03998	0.54	1.14938	0.56663	0.49299	1.04	1.59134	1.23788	0.77789
0.05	1.00125	0.05002	0.04996	0.55	1.15510	0.57815	0.50052	1.05	1.60379	1.25386	0.78181
0.06	1.00180	0.06004	0.05993	0.56	1.16094	0.58973	0.50798	1.06	1.61641	1.26996	0.78566
0.07	1.00245	0.07006	0.06989	0.57	1.16690	0.60137	0.51536	1.07	1.62919	1.28619	0.78946
0.08	1.00320	0.08009	0.07983	0.58	1.17297	0.61307	0.52267	1.08	1.64214	1.30254	0.79320
0.09	1.00405	0.09012	0.08976	0.59	1.17916	0.62483	0.52990	1.09	1.65525	1.31903	0.79688
0.10	1.00500	0.10017	0.09967	0.60	1.18547	0.63665	0.53705	1.10	1.66852	1.33565	0.80050
0.11	1.00606	0.11022	0.10956	0.61	1.19189	0.64854	0.54413	1.11	1.68196	1.35240	0.80406
0.12	1.00721	0.12029	0.11943	0.62	1.19844	0.66049	0.55113	1.12	1.69557	1.36929	0.80757
0.13	1.00846	0.13037	0.12927	0.63	1.20510	0.67251	0.55805	1.13	1.70934	1.38631	0.81102
0.14	1.00982	0.14046	0.13909	0.64	1.21189	0.68459	0.56490	1.14	1.72329	1.40347	0.81441
0.15	1.01127	0.15056	0.14889	0.65	1.21879	0.69675	0.57167	1.15	1.73741	1.42078	0.81775
0.16	1.01283	0.16068	0.15865	0.66	1.22582	0.70897	0.57836	1.16	1.75171	1.43822	0.82104
0.17	1.01448	0.17082	0.16838	0.67	1.23297	0.72126	0.58498	1.17	1.76618	1.45581	0.82427
0.18	1.01624	0.18097	0.17808	0.68	1.24025	0.73363	0.59152	1.18	1.78083	1.47355	0.82745
0.19	1.01810	0.19115	0.18775	0.69	1.24765	0.74607	0.59798	1.19	1.79565	1.49143	0.83058
0.20	1.02007	0.20134	0.19738	0.70	1.25517	0.75858	0.60437	1.20	1.81066	1.50946	0.83365
0.21	1.02213	0.21155	0.20697	0.71	1.26282	0.77117	0.61068	1.21	1.82584	1.52764	0.83668
0.22	1.02430	0.22178	0.21652	0.72	1.27059	0.78384	0.61691	1.22	1.84121	1.54598	0.83965
0.23	1.02657	0.23203	0.22603	0.73	1.27849	0.79659	0.62307	1.23	1.85676	1.56447	0.84258
0.24	1.02894	0.24231	0.23550	0.74	1.28652	0.80941	0.62915	1.24	1.87250	1.58311	0.84546
0.25	1.03141	0.25261	0.24492	0.75	1.29468	0.82232	0.63515	1.25	1.88842	1.60192	0.84828
0.26	1.03399	0.26294	0.25430	0.76	1.30297	0.83530	0.64108	1.26	1.90454	1.62088	0.85106
0.27	1.03667	0.27329	0.26362	0.77	1.31139	0.84838	0.64693	1.27	1.92084	1.64001	0.85380
0.28	1.03946	0.28367	0.27291	0.78	1.31994	0.86153	0.65271	1.28	1.93734	1.65930	0.85648
0.29	1.04235	0.29408	0.28213	0.79	1.32862	0.87478	0.65841	1.29	1.95403	1.67876	0.85913
0.30	1.04534	0.30452	0.29131	0.80	1.33743	0.88811	0.66404	1.30	1.97091	1.69838	0.86172
0.31	1.04844	0.31499	0.30044	0.81	1.34638	0.90152	0.66959	1.31	1.98800	1.71818	0.86428
0.32	1.05164	0.32549	0.30951	0.82	1.35547	0.91503	0.67507	1.32	2.00528	1.73814	0.86678
0.33	1.05495	0.33602	0.31852	0.83	1.36468	0.92863	0.68048	1.33	2.02276	1.75828	0.86925
0.34	1.05836	0.34659	0.32748	0.84	1.37404	0.94233	0.68581	1.34	2.04044	1.77860	0.87167
0.35	1.06188	0.35719	0.33638	0.85	1.38353	0.95612	0.69107	1.35	2.05833	1.79909	0.87405
0.36	1.06550	0.36783	0.34521	0.86	1.39316	0.97000	0.69626	1.36	2.07643	1.81977	0.87639
0.37	1.06923	0.37850	0.35399	0.87	1.40293	0.98398	0.70137	1.37	2.09473	1.84062	0.87869
0.38	1.07307	0.38921	0.36271	0.88	1.41284	0.99806	0.70642	1.38	2.11324	1.86166	0.88095
0.39	1.07702	0.39996	0.37136	0.89	1.42289	1.01224	0.71139	1.39	2.13196	1.88289	0.88317
0.40	1.08107	0.41075	0.37995	0.90	1.43309	1.02652	0.71630	1.40	2.15090	1.90430	0.88535
0.41	1.08523	0.42158	0.38847	0.91	1.44342	1.04090	0.72113	1.41	2.17005	1.92591	0.88749
0.42	1.08950	0.43246	0.39693	0.92	1.45390	1.05539	0.72590	1.42	2.18942	1.94770	0.88960
0.43	1.09388	0.44337	0.40532	0.93	1.46453	1.06998	0.73059	1.43	2.20900	1.96970	0.89167
0.44	1.09837	0.45434	0.41364	0.94	1.47530	1.08468	0.73522	1.44	2.22881	1.99188	0.89370
0.45	1.10297	0.46534	0.42190	0.95	1.48623	1.09948	0.73978	1.45	2.24884	2.01427	0.89569
0.46	1.10768	0.47640	0.43008	0.96	1.49729	1.11440	0.74428	1.46	2.26910	2.03686	0.89765
0.47	1.11250	0.48750	0.43820	0.97	1.50851	1.12943	0.74870	1.47	2.28958	2.05965	0.89958
0.48	1.11743	0.49865	0.44624	0.98	1.51988	1.14457	0.75307	1.48	2.31029	2.08265	0.90147
0.49	1.12247	0.50984	0.45422	0.99	1.53141	1.15983	0.75736	1.49	2.33123	2.10566	0.90332
0.50	1.12763	0.52110	0.46212	1.00	1.54308	1.17520	0.76159	1.50	2.35241	2.12928	0.90515

TABLE 1A. Lanchester-Clifford-Schläfli Functions $F_\alpha(x)$, $H_{1-\alpha}(x)$, and $T_\alpha(x)$ for $\alpha = 1/2$ and x from 0.00 to 1.50.

$\alpha = 1/2$

x	$F_{1/2}(x)$	$H_{1/2}(x)$	$T_{1/2}(x)$
1.50	2.35241	2.12928	0.90515
1.51	2.37382	2.15291	0.90694
1.52	2.39547	2.17676	0.90870
1.53	2.41736	2.20082	0.91042
1.54	2.43949	2.22510	0.91212
1.55	2.46186	2.24961	0.91379
1.56	2.48448	2.27434	0.91542
1.57	2.50735	2.29930	0.91703
1.58	2.53047	2.32449	0.91860
1.59	2.55384	2.34991	0.92015
1.60	2.57746	2.37557	0.92167
1.61	2.60135	2.40146	0.92316
1.62	2.62549	2.42760	0.92462
1.63	2.64990	2.45397	0.92606
1.64	2.67457	2.48059	0.92747
1.65	2.69951	2.50746	0.92886
1.66	2.72472	2.53459	0.93022
1.67	2.75021	2.56196	0.93155
1.68	2.77596	2.58959	0.93286
1.69	2.80200	2.61748	0.93415
1.70	2.82832	2.64563	0.93541
1.71	2.85491	2.67405	0.93665
1.72	2.88180	2.70273	0.93786
1.73	2.90897	2.73168	0.93906
1.74	2.93643	2.76091	0.94023
1.75	2.96419	2.79041	0.94138
1.76	2.99224	2.82020	0.94250
1.77	3.02059	2.85026	0.94361
1.78	3.04925	2.88061	0.94470
1.79	3.07821	2.91125	0.94576
1.80	3.10747	2.94217	0.94681
1.81	3.13705	2.97340	0.94783
1.82	3.16694	3.00492	0.94884
1.83	3.19715	3.03674	0.94983
1.84	3.22768	3.06886	0.95080
1.85	3.25853	3.10129	0.95175
1.86	3.28970	3.13403	0.95268
1.87	3.32121	3.16709	0.95359
1.88	3.35305	3.20046	0.95449
1.89	3.38522	3.23415	0.95537
1.90	3.41773	3.26816	0.95624
1.91	3.45058	3.30250	0.95709
1.92	3.48378	3.33718	0.95792
1.93	3.51733	3.37218	0.95873
1.94	3.55123	3.40752	0.95953
1.95	3.58548	3.44321	0.96032
1.96	3.62009	3.47923	0.96109
1.97	3.65507	3.51561	0.96185
1.98	3.69041	3.55234	0.96259
1.99	3.72611	3.58942	0.96331
2.00	3.76220	3.62686	0.96403

x	$F_{1/2}(x)$	$H_{1/2}(x)$	$T_{1/2}(x)$
2.0	3.76220	3.62686	0.96403
2.1	4.14431	4.02186	0.97045
2.2	4.56791	4.45711	0.97574
2.3	5.03722	4.93696	0.98010
2.4	5.55695	5.46623	0.98367
2.5	6.13229	6.05020	0.98661
2.6	6.76901	6.69473	0.98903
2.7	7.47347	7.40626	0.99101
2.8	8.25273	8.19192	0.99263
2.9	9.11458	9.05956	0.99396
3.0	10.06766	10.01787	0.99505
3.1	11.12150	11.07645	0.99595
3.2	12.28665	12.24588	0.99668
3.3	13.57476	13.53788	0.99728
3.4	14.99874	14.96536	0.99777
3.5	16.57282	16.54263	0.99818
3.6	18.31278	18.28546	0.99851
3.7	20.23601	20.21129	C.99878
3.8	22.36178	22.33941	0.99900
3.9	24.71135	24.69110	0.99918
4.0	27.30823	27.28992	0.99933
4.1	30.17843	30.16186	0.99945
4.2	33.35066	33.33567	0.99955
4.3	36.85668	36.84311	0.99963
4.4	40.73157	40.71930	0.99970
4.5	45.01412	45.00301	0.99975
4.6	49.74718	49.73713	0.99980
4.7	54.97813	54.96904	0.99983
4.8	60.75932	60.75109	0.99986
4.9	67.14861	67.14117	0.99989
5.0	74.20995	74.20321	0.99991
5.1	82.01400	82.00791	0.99993
5.2	90.63888	90.63336	0.99994
5.3	100.17090	100.16591	0.99995
5.4	110.70547	110.70095	0.99996
5.5	122.34801	122.34392	0.99997
5.6	135.21505	135.21135	C.99997
5.7	149.43537	149.43203	0.99998
5.8	165.15129	165.14827	0.99998
5.9	182.52010	182.51736	C.99998
6.0	201.71564	201.71316	0.99999

x	$F_{1/2}(x)$	$H_{1/2}(x)$	$T_{1/2}(x)$
6.0	201.71564	201.71316	0.99999
6.1	222.93001	222.92776	0.99999
6.2	246.37554	246.37351	0.99999
6.3	272.28687	272.28504	0.99999
6.4	300.92335	300.92169	0.99999
6.5	332.57157	332.57006	1.00000
6.6	367.54827	367.54691	1.00000
6.7	406.20353	406.20230	1.00000
6.8	448.92420	448.92309	1.00000
6.9	496.13786	496.13685	1.00000
7.0	548.31704	548.31612	1.00000
7.1	605.98395	605.98312	1.00000
7.2	669.71576	669.71501	1.00000
7.3	740.15030	740.14963	1.00000
7.4	817.99252	817.99191	1.00000
7.5	904.02148	904.02093	1.00000
7.6	999.09820	999.09770	1.00000
7.7	1104.17422	1104.17377	1.00000
7.8	1220.30119	1220.30078	1.00000
7.9	1348.64135	1348.64098	1.00000
8.0	1490.47916	1490.47883	1.00000
8.1	1647.23419	1647.23389	1.00000
8.2	1820.47529	1820.47502	1.00000
8.3	2011.93632	2011.93607	1.00000
8.4	2223.53349	2223.53326	1.00000
8.5	2457.38452	2457.38432	1.00000
8.6	2715.82989	2715.82970	1.00000
8.7	3001.45619	3001.45603	1.00000
8.8	3317.12208	3317.12193	1.00000
8.9	3665.98684	3665.98670	1.00000
9.0	4051.54203	4051.54190	1.00000
9.1	4477.64641	4477.64630	1.00000
9.2	4948.56458	4948.56448	1.00000
9.3	5469.00965	5469.00956	1.00000
9.4	6044.19041	6044.19032	1.00000
9.5	6679.86345	6679.86338	1.00000
9.6	7382.39082	7382.39075	1.00000
9.7	8158.80363	8158.80357	1.00000
9.8	9016.87249	9016.87244	1.00000
9.9	9965.18524	9965.18519	1.00000
10.0	11013.23292	11013.23287	1.00000

TABLE 1B. Lanchester-Clifford-Schläfli Functions $F_\alpha(x)$, $H_{1-\alpha}(x)$, and $T_\alpha(x)$ for $\alpha = 1/2$ and x from 1.50 to 10.0.

$\alpha = 1/3$

x	$F_{1/3}(x)$	$H_{2/3}(x)$	$T_{1/3}(x)$	x	$F_{1/3}(x)$	$H_{2/3}(x)$	$T_{1/3}(x)$	x	$F_{1/3}(x)$	$H_{2/3}(x)$	$T_{1/3}(x)$
0.0	1.00000	0.0	0.0	C.50	1.19193	0.24520	0.20571	1.00	1.82287	0.68885	0.37789
0.01	1.00008	0.00128	0.00128	0.51	1.19988	0.25214	0.21013	1.01	1.84096	0.70005	0.38026
0.02	1.00030	0.00323	0.00323	0.52	1.20799	0.25914	0.21452	1.02	1.85929	0.71135	0.38259
0.03	1.00068	0.00555	0.00555	0.53	1.21628	0.26622	0.21888	1.03	1.87787	0.72276	0.38488
0.04	1.00120	0.00815	0.00814	C.54	1.22474	0.27337	0.22321	1.04	1.89670	0.73428	0.38714
0.05	1.00188	0.01097	0.01095	0.55	1.23338	0.28059	0.22750	1.05	1.91578	0.74592	0.38935
0.06	1.00270	0.01399	0.01395	0.56	1.24219	0.28789	0.23176	1.06	1.93511	0.75766	0.39153
0.07	1.00368	0.01719	0.01712	0.57	1.25118	0.29525	0.23598	1.07	1.95469	0.76952	0.39368
0.08	1.00480	0.02054	0.02044	0.58	1.26035	0.30269	0.24016	1.08	1.97453	0.78149	0.39578
C.09	1.00608	0.02404	0.02389	0.59	1.26970	0.31020	0.24431	1.09	1.99463	0.79357	0.39785
0.10	1.00751	0.02767	0.02747	0.60	1.27923	0.31779	0.24842	1.10	2.01500	0.80578	0.39989
0.11	1.00909	0.03143	0.03115	0.61	1.28894	0.32545	0.25249	1.11	2.03562	0.81810	0.40189
0.12	1.01081	0.03531	0.03493	C.62	1.29883	0.33318	0.25652	1.12	2.05651	0.83054	0.40386
0.13	1.01270	0.03930	0.03881	0.63	1.30891	0.34099	0.26052	1.13	2.07767	0.84310	0.40579
0.14	1.01473	0.04340	0.04277	0.64	1.31917	0.34888	0.26447	1.14	2.09910	0.85578	0.40769
0.15	1.01691	0.04760	0.04681	0.65	1.32962	0.35684	0.26838	1.15	2.12081	0.86858	0.40955
0.16	1.01925	0.05191	0.05093	0.66	1.34025	0.36488	0.27225	1.16	2.14279	0.88151	0.41138
0.17	1.02173	0.05630	0.05511	0.67	1.35107	0.37299	0.27607	1.17	2.16504	0.89456	0.41318
C.18	1.02437	0.06079	0.05935	0.68	1.36208	0.38119	0.27985	1.18	2.18758	0.90774	0.41495
0.19	1.02717	0.06537	0.06365	C.69	1.37329	0.38946	0.28360	1.19	2.21040	0.92105	0.41669
0.20	1.03011	0.07004	0.06799	0.70	1.38468	0.39781	0.28729	1.20	2.23350	0.93449	0.41840
0.21	1.03321	0.07480	0.07239	C.71	1.39627	0.40624	0.29095	1.21	2.25690	0.94806	0.42007
0.22	1.03646	0.07963	0.07683	0.72	1.40805	0.41475	0.29456	1.22	2.28058	0.96176	0.42172
0.23	1.03987	0.08455	0.08131	C.73	1.42003	0.42334	0.29812	1.23	2.30456	0.97560	0.42333
0.24	1.04343	0.08955	0.08583	0.74	1.43220	0.43202	0.30164	1.24	2.32883	0.98957	0.42492
0.25	1.04715	0.09463	0.09037	C.75	1.44457	0.44077	0.30512	1.25	2.35340	1.00367	0.42648
0.26	1.05102	0.09979	0.09494	0.76	1.45715	0.44961	0.30856	1.26	2.37827	1.01792	0.42801
0.27	1.05505	0.10502	0.09954	C.77	1.46992	0.45853	0.31195	1.27	2.40345	1.03230	0.42951
0.28	1.05923	0.11033	0.10416	0.78	1.48290	0.46754	0.31529	1.28	2.42893	1.04683	0.43098
0.29	1.06357	0.11571	0.10880	0.79	1.49608	0.47664	0.31859	1.29	2.45473	1.06150	0.43243
0.30	1.06807	0.12117	0.11345	C.80	1.50947	0.48582	0.32185	1.30	2.48083	1.07631	0.43385
0.31	1.07273	0.12670	0.11811	C.81	1.52306	0.49508	0.32506	1.31	2.50725	1.09127	0.43525
0.32	1.07754	0.13230	0.12278	0.82	1.53686	0.50444	0.32822	1.32	2.53399	1.10638	0.43661
C.33	1.08251	0.13798	0.12746	0.83	1.55088	0.51388	0.33135	1.33	2.56105	1.12163	0.43796
0.34	1.08764	0.14372	0.13214	0.84	1.56510	0.52341	0.33443	1.34	2.58844	1.13704	0.43928
C.35	1.09293	0.14954	0.13683	0.85	1.57954	0.53304	0.33746	1.35	2.61615	1.15260	0.44057
0.36	1.09839	0.15543	0.14151	0.86	1.59419	0.54275	0.34045	1.36	2.64419	1.16831	0.44184
0.37	1.10400	0.16139	0.14618	0.87	1.60906	0.55256	0.34340	1.37	2.67257	1.18418	0.44309
C.38	1.10977	0.16742	0.15086	0.88	1.62415	0.56246	0.34631	1.38	2.70128	1.20020	0.44431
0.39	1.11571	0.17352	0.15552	0.89	1.63946	0.57245	0.34917	1.39	2.73034	1.21639	0.44551
C.40	1.12181	0.17968	0.16017	0.90	1.65499	0.58254	0.35199	1.40	2.75973	1.23273	0.44668
0.41	1.12807	0.18592	0.16481	0.91	1.67074	0.59272	0.35477	1.41	2.78947	1.24924	0.44784
0.42	1.13450	0.19223	0.16944	0.92	1.68672	0.60300	0.35750	1.42	2.81957	1.26591	0.44897
0.43	1.14109	0.19861	0.17405	0.93	1.70292	0.61338	0.36019	1.43	2.85001	1.28274	0.45008
0.44	1.14785	0.20505	0.17864	C.94	1.71936	0.62386	0.36284	1.44	2.88081	1.29975	0.45117
C.45	1.15478	0.21157	0.18321	0.95	1.73602	0.63443	0.36545	1.45	2.91197	1.31692	0.45224
0.46	1.16187	0.21815	0.18776	C.96	1.75292	0.64511	0.36802	1.46	2.94350	1.33427	0.45329
C.47	1.16913	0.22481	0.19229	0.97	1.77005	0.65589	0.37055	1.47	2.97539	1.35179	0.45432
0.48	1.17656	0.23154	0.19679	C.98	1.78742	0.66677	0.37304	1.48	3.00765	1.36948	0.45533
0.49	1.18416	0.23833	0.20127	C.99	1.80503	0.67776	0.37548	1.49	3.04029	1.38735	0.45632
0.50	1.19193	0.24520	0.20571	1.00	1.82287	0.68885	0.37789	1.50	3.07330	1.40540	0.45729

TABLE 2A. Lanchester-Clifford-Schläfli Functions $F_\alpha(x)$, $H_{1-\alpha}(x)$, and $T_\alpha(x)$ for $\alpha = 1/3$ and x from 0.00 to 1.50.

$\alpha = 1/3$

x	$F_{1/3}(x)$	$H_{2/3}(x)$	$T_{1/3}(x)$
1.50	3.07330	1.40540	0.45729
1.51	3.10669	1.42363	0.45825
1.52	3.14047	1.44204	0.45918
1.53	3.17464	1.46064	0.46010
1.54	3.20920	1.47943	0.46100
1.55	3.24416	1.49840	0.46188
1.56	3.27951	1.51757	0.46274
1.57	3.31527	1.53692	0.46359
1.58	3.35144	1.55648	0.46442
1.59	3.38802	1.57622	0.46524
1.60	3.42501	1.59617	0.46603
1.61	3.46243	1.61632	0.46682
1.62	3.50027	1.63668	0.46759
1.63	3.53854	1.65724	0.46834
1.64	3.57724	1.67800	0.46908
1.65	3.61637	1.69898	0.46980
1.66	3.65595	1.72017	0.47051
1.67	3.69598	1.74158	0.47121
1.68	3.73645	1.76320	0.47189
1.69	3.77738	1.78504	0.47256
1.70	3.81877	1.80710	0.47322
1.71	3.86062	1.82939	0.47386
1.72	3.90294	1.85191	0.47449
1.73	3.94573	1.87465	0.47511
1.74	3.98900	1.89762	0.47571
1.75	4.03275	1.92083	0.47631
1.76	4.07699	1.94428	0.47689
1.77	4.12172	1.96796	0.47746
1.78	4.16695	1.99189	0.47802
1.79	4.21267	2.01606	0.47857
1.80	4.25891	2.04048	0.47911
1.81	4.30566	2.06514	0.47963
1.82	4.35292	2.09006	0.48015
1.83	4.40071	2.11523	0.48066
1.84	4.44903	2.14067	0.48115
1.85	4.49787	2.16636	0.48164
1.86	4.54726	2.19231	0.48212
1.87	4.59719	2.21853	0.48258
1.88	4.64767	2.24502	0.48304
1.89	4.69870	2.27178	0.48349
1.90	4.75030	2.29882	0.48393
1.91	4.80246	2.32613	0.48436
1.92	4.85519	2.35373	0.48479
1.93	4.90850	2.38160	0.48520
1.94	4.96239	2.40977	0.48561
1.95	5.01688	2.43822	0.48600
1.96	5.07195	2.46697	0.48639
1.97	5.12763	2.49601	0.48678
1.98	5.18392	2.52535	0.48715
1.99	5.24082	2.55499	0.48752
2.00	5.29834	2.58494	0.48788

x	$F_{1/3}(x)$	$H_{2/3}(x)$	$T_{1/3}(x)$
2.0	5.29834	2.58494	0.48788
2.1	5.90906	2.90196	0.49110
2.2	6.58916	3.25333	0.49374
2.3	7.34609	3.64286	0.49589
2.4	8.18812	4.07480	0.49765
2.5	9.12445	4.55385	0.49908
2.6	10.16526	5.08521	0.50025
2.7	11.32190	5.67464	0.50121
2.8	12.60691	6.32854	0.50199
2.9	14.03424	7.05399	0.50263
3.0	15.61932	7.85882	0.50315
3.1	17.37931	8.75173	0.50357
3.2	19.33319	9.74236	0.50392
3.3	21.50203	10.84137	0.50420
3.4	23.90917	12.06058	0.50443
3.5	26.58048	13.41311	0.50462
3.6	29.54465	14.91345	0.50478
3.7	32.83344	16.57770	0.50490
3.8	36.48208	18.42367	0.50501
3.9	40.52959	20.47111	0.50509
4.0	45.01920	22.74186	0.50516
4.1	49.99881	25.26018	0.50522
4.2	55.52151	28.05289	0.50526
4.3	61.64609	31.14972	0.50530
4.4	68.43768	34.58361	0.50533
4.5	75.96846	38.39105	0.50536
4.6	84.31835	42.61245	0.50538
4.7	93.57590	47.29256	0.50539
4.8	103.83919	52.48098	0.50541
4.9	115.21685	58.23262	0.50542
5.0	127.82922	64.60830	0.50543
5.1	141.80955	71.67541	0.50543
5.2	157.30546	79.50854	0.50544
5.3	174.48042	88.19033	0.50545
5.4	193.51547	97.81231	0.50545
5.5	214.61111	108.47582	0.50545
5.6	237.98938	120.29309	0.50546
5.7	263.89617	133.38841	0.50546
5.8	292.60376	147.89943	0.50546
5.9	324.41368	163.97856	0.50546
6.0	359.65982	181.79456	0.50546

x	$F_{1/3}(x)$	$H_{2/3}(x)$	$T_{1/3}(x)$
6.0	359.65982	181.79456	0.50546
6.1	398.71187	201.53431	0.50546
6.2	441.97922	223.40475	0.50546
6.3	489.91515	247.63498	0.50547
6.4	543.02156	274.47872	0.50547
6.5	601.85414	304.21683	0.50547
6.6	667.02816	337.16033	0.50547
6.7	739.22482	373.65355	0.50547
6.8	819.19834	414.07770	0.50547
6.9	907.78370	458.85486	0.50547
7.0	1005.90533	508.45229	0.50547
7.1	1114.58667	563.38731	0.50547
7.2	1234.96073	624.23262	0.50547
7.3	1368.28181	691.62223	0.50547
7.4	1515.93846	766.25801	0.50547
7.5	1679.46784	848.91694	0.50547
7.6	1860.57158	940.45914	0.50547
7.7	2061.13333	1041.83675	0.50547
7.8	2283.23826	1154.10374	0.50547
7.9	2529.19452	1278.42680	0.50547
8.0	2801.55706	1416.09741	0.50547
8.1	3103.15401	1568.54507	0.50547
8.2	3437.11583	1737.35213	0.50547
8.3	3806.90755	1924.27007	0.50547
8.4	4216.36450	2131.23751	0.50547
8.5	4669.73184	2360.40025	0.50547
8.6	5171.70827	2614.13334	0.50547
8.7	5727.49440	2895.06550	0.50547
8.8	6342.84633	3206.10628	0.50547
8.9	7024.13486	3550.47590	0.50547
9.0	7778.41109	3931.73848	0.50547
9.1	8613.47893	4353.83863	0.50547
9.2	9537.97538	4821.14209	0.50547
9.3	10561.45936	5338.48059	0.50547
9.4	11694.51000	5911.20154	0.50547
9.5	12948.83537	6545.22299	0.50547
9.6	14337.39281	7247.09447	0.50547
9.7	15874.52207	8024.06427	0.50547
9.8	17576.09265	8884.15390	0.50547
9.9	19459.66675	9836.24050	0.50547
10.0	21544.67965	10890.14799	0.50547

TABLE 2B. Lanchester-Clifford-Schläfli Functions $F_\alpha(x)$, $H_{1-\alpha}(x)$, and $T_\alpha(x)$ for $\alpha = 1/3$ and x from 1.50 to 10.0.

$\alpha = 2/3$

x	$F_{2/3}(x)$	$H_{1/3}(x)$	$T_{2/3}(x)$	x	$F_{2/3}(x)$	$H_{1/3}(x)$	$T_{2/3}(x)$	x	$F_{2/3}(x)$	$H_{1/3}(x)$	$T_{2/3}(x)$
0.0	1.00000	0.0	0.0	0.50	1.09552	1.24711	1.13837	1.00	1.40402	2.26370	1.61230
0.01	1.00004	0.08772	0.08772	0.51	1.09946	1.26603	1.15151	1.01	1.41275	2.28674	1.61864
0.02	1.00015	0.13926	0.13924	0.52	1.10347	1.28495	1.16446	1.02	1.42160	2.30994	1.62488
0.03	1.00034	0.18250	0.18244	0.53	1.10758	1.30389	1.17724	1.03	1.43056	2.33328	1.63103
0.04	1.00060	0.22111	0.22098	0.54	1.11176	1.32283	1.18985	1.04	1.43964	2.35679	1.63708
0.05	1.00094	0.25662	0.25638	0.55	1.11604	1.34178	1.20228	1.05	1.44882	2.38046	1.64303
0.06	1.00135	0.28984	0.28945	0.56	1.12039	1.36076	1.21454	1.06	1.45813	2.40429	1.64889
0.07	1.00184	0.32129	0.32070	0.57	1.12484	1.37976	1.22663	1.07	1.46755	2.42828	1.65465
0.08	1.00240	0.35130	0.35046	0.58	1.12937	1.39879	1.23856	1.08	1.47709	2.45245	1.66033
0.09	1.00304	0.38012	0.37897	0.59	1.13398	1.41784	1.25032	1.09	1.48674	2.47678	1.66591
0.10	1.00375	0.40793	0.40640	0.60	1.13869	1.43694	1.26192	1.10	1.49652	2.50129	1.67140
0.11	1.00454	0.43486	0.43289	0.61	1.14348	1.45606	1.27337	1.11	1.50641	2.52597	1.67681
0.12	1.00541	0.46103	0.45855	0.62	1.14836	1.47523	1.28465	1.12	1.51643	2.55083	1.68213
0.13	1.00635	0.48653	0.48346	0.63	1.15332	1.49445	1.29578	1.13	1.52656	2.57587	1.68736
0.14	1.00736	0.51142	0.50769	0.64	1.15838	1.51371	1.30675	1.14	1.53682	2.60109	1.69251
0.15	1.00845	0.53579	0.53130	0.65	1.16352	1.53302	1.31757	1.15	1.54721	2.62650	1.69758
0.16	1.00962	0.55967	0.55434	0.66	1.16876	1.55239	1.32824	1.16	1.55772	2.65210	1.70256
0.17	1.01086	0.58311	0.57685	0.67	1.17409	1.57181	1.33875	1.17	1.56835	2.67789	1.70746
0.18	1.01218	0.60616	0.59886	0.68	1.17950	1.59130	1.34913	1.18	1.57911	2.70388	1.71228
0.19	1.01357	0.62884	0.62042	0.69	1.18501	1.61084	1.35935	1.19	1.58999	2.73006	1.71702
0.20	1.01505	0.65119	0.64154	0.70	1.19061	1.63046	1.36943	1.20	1.60101	2.75644	1.72169
0.21	1.01659	0.67323	0.66225	0.71	1.19630	1.65014	1.37937	1.21	1.61215	2.78302	1.72627
0.22	1.01822	0.69500	0.68256	0.72	1.20208	1.66990	1.38917	1.22	1.62343	2.80981	1.73079
0.23	1.01992	0.71650	0.70251	0.73	1.20796	1.68973	1.39883	1.23	1.63484	2.83680	1.73522
0.24	1.02169	0.73777	0.72211	0.74	1.21393	1.70964	1.40835	1.24	1.64637	2.86401	1.73959
0.25	1.02355	0.75882	0.74136	0.75	1.21999	1.72963	1.41774	1.25	1.65805	2.89143	1.74388
0.26	1.02548	0.77966	0.76029	0.76	1.22615	1.74970	1.42699	1.26	1.66985	2.91906	1.74809
0.27	1.02749	0.80032	0.77891	0.77	1.23241	1.76987	1.43610	1.27	1.68179	2.94691	1.75224
0.28	1.02957	0.82080	0.79722	0.78	1.23876	1.79012	1.44509	1.28	1.69387	2.97499	1.75632
0.29	1.03174	0.84112	0.81525	0.79	1.24521	1.81046	1.45394	1.29	1.70609	3.00328	1.76033
0.30	1.03398	0.86129	0.83299	0.80	1.25175	1.83090	1.46267	1.30	1.71844	3.03181	1.76427
0.31	1.03630	0.88133	0.85046	0.81	1.25840	1.85143	1.47127	1.31	1.73094	3.06056	1.76815
0.32	1.03870	0.90124	0.86766	0.82	1.26514	1.87207	1.47974	1.32	1.74358	3.08955	1.77196
0.33	1.04117	0.92103	0.88461	0.83	1.27198	1.89281	1.48809	1.33	1.75635	3.11877	1.77571
0.34	1.04373	0.94071	0.90130	0.84	1.27892	1.91365	1.49631	1.34	1.76928	3.14823	1.77939
0.35	1.04636	0.96030	0.91775	0.85	1.28595	1.93461	1.50441	1.35	1.78234	3.17794	1.78301
0.36	1.04907	0.97979	0.93396	0.86	1.29309	1.95567	1.51240	1.36	1.79556	3.20788	1.78657
0.37	1.05187	0.99921	0.94994	0.87	1.30034	1.97685	1.52026	1.37	1.80892	3.23807	1.79006
0.38	1.05474	1.01855	0.96569	0.88	1.30768	1.99815	1.52801	1.38	1.82242	3.26852	1.79350
0.39	1.05769	1.03782	0.98121	0.89	1.31513	2.01956	1.53564	1.39	1.83608	3.29921	1.79688
0.40	1.06072	1.05703	0.99652	0.90	1.32268	2.04110	1.54316	1.40	1.84989	3.33017	1.80020
0.41	1.06384	1.07619	1.01161	0.91	1.33033	2.06275	1.55056	1.41	1.86385	3.36138	1.80346
0.42	1.06703	1.09530	1.02649	0.92	1.33809	2.08454	1.55785	1.42	1.87796	3.39285	1.80667
0.43	1.07030	1.11436	1.04116	0.93	1.34595	2.10646	1.56503	1.43	1.89223	3.42459	1.80982
0.44	1.07366	1.13339	1.05563	0.94	1.35392	2.12850	1.57210	1.44	1.90665	3.45660	1.81291
0.45	1.07710	1.15239	1.06990	0.95	1.36200	2.15068	1.57906	1.45	1.92123	3.48888	1.81596
0.46	1.08062	1.17137	1.08398	0.96	1.37019	2.17300	1.58592	1.46	1.93597	3.52143	1.81895
0.47	1.08422	1.19032	1.09786	0.97	1.37848	2.19546	1.59267	1.47	1.95087	3.55426	1.82188
0.48	1.08790	1.20926	1.11155	0.98	1.38688	2.21806	1.59931	1.48	1.96593	3.58737	1.82477
0.49	1.09167	1.22819	1.12505	0.99	1.39540	2.24081	1.60586	1.49	1.98115	3.62076	1.82761
0.50	1.09552	1.24711	1.13837	1.00	1.40402	2.26370	1.61230	1.50	1.99654	3.65444	1.83039

TABLE 3A. Lanchester-Clifford-Schläfli Functions $F_\alpha(x)$, $H_{1-\alpha}(x)$, and $T_\alpha(x)$ for $\alpha = 2/3$ and x from 0.00 to 1.50.

$\alpha = 2/3$

x	$F_{2/3}(x)$	$H_{1/3}(x)$	$T_{2/3}(x)$
1.50	1.99654	3.65444	1.83039
1.51	2.01209	3.68842	1.83313
1.52	2.02781	3.72268	1.83582
1.53	2.04369	3.75724	1.83846
1.54	2.05975	3.79211	1.84105
1.55	2.07598	3.82727	1.84360
1.56	2.09237	3.86274	1.84610
1.57	2.10895	3.89853	1.84856
1.58	2.12570	3.93462	1.85098
1.59	2.14262	3.97103	1.85335
1.60	2.15973	4.00777	1.85568
1.61	2.17701	4.04483	1.85797
1.62	2.19448	4.08221	1.86022
1.63	2.21213	4.11993	1.86243
1.64	2.22996	4.15798	1.86459
1.65	2.24798	4.19636	1.86672
1.66	2.26619	4.23509	1.86881
1.67	2.28459	4.27417	1.87087
1.68	2.30318	4.31360	1.87288
1.65	2.32197	4.35337	1.87486
1.70	2.34095	4.39351	1.87681
1.71	2.36012	4.43400	1.87872
1.72	2.37950	4.47486	1.88059
1.73	2.39907	4.51609	1.88243
1.74	2.41885	4.55768	1.88424
1.75	2.43883	4.59966	1.88601
1.76	2.45901	4.64201	1.88775
1.77	2.47940	4.68475	1.88946
1.78	2.50001	4.72787	1.89114
1.79	2.52082	4.77139	1.89279
1.80	2.54185	4.81530	1.89441
1.81	2.56309	4.85961	1.89600
1.82	2.58455	4.90433	1.89756
1.83	2.60623	4.94945	1.89909
1.84	2.62813	4.99499	1.90059
1.85	2.65025	5.04094	1.90207
1.86	2.67259	5.08732	1.90351
1.87	2.69517	5.13412	1.90493
1.88	2.71797	5.18135	1.90633
1.89	2.74100	5.22901	1.90770
1.90	2.76427	5.27711	1.90904
1.91	2.78777	5.32565	1.91036
1.92	2.81152	5.37465	1.91165
1.93	2.83550	5.42409	1.91293
1.94	2.85972	5.47399	1.91417
1.95	2.88418	5.52435	1.91540
1.96	2.90890	5.57518	1.91660
1.97	2.93386	5.62648	1.91778
1.98	2.95907	5.67825	1.91893
1.99	2.98453	5.73051	1.92007
2.00	3.01025	5.78325	1.92118

x	$F_{2/3}(x)$	$H_{1/3}(x)$	$T_{2/3}(x)$
2.0	3.01025	5.78325	1.92118
2.1	3.28205	6.33842	1.93124
2.2	3.58216	6.94779	1.93955
2.3	3.91334	7.61702	1.94643
2.4	4.27863	8.35230	1.95209
2.5	4.68142	9.16046	1.95677
2.6	5.12542	10.04900	1.96062
2.7	5.61474	11.02615	1.96379
2.8	6.15390	12.10099	1.96639
2.9	6.74791	13.28351	1.96854
3.0	7.40228	14.58471	1.97030
3.1	8.12310	16.01669	1.97175
3.2	8.91707	17.59281	1.97294
3.3	9.79160	19.32776	1.97391
3.4	10.75486	21.23775	1.97471
3.5	11.81582	23.34063	1.97537
3.6	12.98444	25.65607	1.97591
3.7	14.27164	28.20577	1.97635
3.8	15.68949	31.01364	1.97671
3.9	17.25130	34.10602	1.97701
4.0	18.97175	37.51199	1.97726
4.1	20.86700	41.26358	1.97746
4.2	22.95492	45.39611	1.97762
4.3	25.25517	49.94852	1.97775
4.4	27.78944	54.96375	1.97786
4.5	30.58167	60.48916	1.97795
4.6	33.65824	66.57697	1.97803
4.7	37.04825	73.28475	1.97809
4.8	40.78379	80.67601	1.97814
4.9	44.90025	88.82076	1.97818
5.0	49.43666	97.79625	1.97821
5.1	54.43607	107.68763	1.97824
5.2	59.94596	118.58886	1.97826
5.3	66.01869	130.60352	1.97828
5.4	72.71202	143.84591	1.97830
5.5	80.08966	158.44205	1.97831
5.6	88.22187	174.53096	1.97832
5.7	97.18616	192.26599	1.97833
5.8	107.06803	211.81628	1.97833
5.9	117.96178	233.36842	1.97834
6.0	129.97145	257.12820	1.97834

x	$F_{2/3}(x)$	$H_{1/3}(x)$	$T_{2/3}(x)$
6.0	129.97145	257.12820	1.97834
6.1	143.21179	283.32268	1.97835
6.2	157.80940	312.20230	1.97835
6.3	173.90393	344.04335	1.97835
6.4	191.64943	379.15061	1.97836
6.5	211.21588	417.86035	1.97836
6.6	232.79078	460.54351	1.97836
6.7	256.58098	507.60934	1.97836
6.8	282.81472	559.50936	1.97836
6.9	311.74379	616.74170	1.97836
7.0	343.64598	679.85596	1.97836
7.1	378.82779	749.45850	1.97836
7.2	417.62741	826.21836	1.97836
7.3	460.41794	910.87370	1.97836
7.4	507.61109	1004.23900	1.97836
7.5	559.66109	1107.21292	1.97836
7.6	617.06916	1220.78706	1.97836
7.7	680.38036	1346.05555	1.97836
7.8	750.22894	1484.22569	1.97836
7.9	827.26427	1636.62967	1.97836
8.0	912.23739	1804.73751	1.97836
8.1	1005.96825	1990.17133	1.97836
8.2	1109.36164	2194.72114	1.97836
8.3	1223.41598	2420.36220	1.97836
8.4	1349.23309	2669.27429	1.97836
8.5	1488.02882	2943.86283	1.97836
8.6	1641.14495	3246.78232	1.97836
8.7	1810.06220	3580.96219	1.97836
8.8	1996.41466	3949.63526	1.97836
8.9	2202.00570	4356.36923	1.97836
9.0	2428.82550	4805.10143	1.97836
9.1	2679.07046	5300.17714	1.97836
9.2	2955.[illegible]	5846.39194	1.97836
9.3	3259.40315	6449.03839	1.97836
9.4	3595.87862	7113.95767	1.97836
9.5	3966.70960	7847.59643	1.97836
9.6	4375.87251	8657.06970	1.97836
9.7	4827.33666	9550.23026	1.97836
9.8	5325.48299	10535.74517	1.97836
9.9	5875.14679	11623.18037	1.97836
10.0	6481.66483	12823.09399	1.97836

TABLE 3B. Lanchester-Clifford-Schläfli Functions $F_\alpha(x)$, $H_{1-\alpha}(x)$, and $T_\alpha(x)$ for $\alpha = 2/3$ and x from 1.50 to 10.0.

α = 1/4

x	$F_{1/4}(x)$	$H_{3/4}(x)$	$T_{1/4}(x)$	x	$F_{1/4}(x)$	$H_{3/4}(x)$	$T_{1/4}(x)$	x	$F_{1/4}(x)$	$H_{3/4}(x)$	$T_{1/4}(x)$
C.0	1.00000	0.0	0.0	0.50	1.25631	0.17269	0.13746	1.00	2.10378	0.54188	0.25757
0.01	1.00010	0.00047	0.00047	0.51	1.26693	0.17815	0.14061	1.01	2.12817	0.55153	0.25916
0.02	1.00040	0.00133	0.00133	0.52	1.27779	0.18368	0.14375	1.02	2.15290	0.56129	0.26071
0.03	1.00090	0.00245	0.00245	0.53	1.28887	0.18928	0.14686	1.03	2.17796	0.57115	0.26224
C.04	1.00160	0.00377	0.00377	0.54	1.30020	0.19496	0.14995	1.04	2.20337	0.58111	0.26374
0.05	1.00250	0.00527	0.00526	0.55	1.31175	0.20071	0.15301	1.05	2.22912	0.59118	0.26521
0.06	1.00360	0.00693	0.00691	0.56	1.32355	0.20653	0.15604	1.06	2.25522	0.60136	0.26665
0.07	1.00490	0.00874	0.00869	0.57	1.33558	0.21242	0.15905	1.07	2.28166	0.61165	0.26807
0.08	1.00640	0.01068	0.01061	0.58	1.34786	0.21839	0.16203	1.08	2.30846	0.62205	0.26946
0.09	1.00811	0.01274	0.01264	0.59	1.36037	0.22443	0.16498	1.09	2.33561	0.63256	0.27083
0.10	1.01001	0.01493	0.01478	0.60	1.37313	0.23054	0.16789	1.10	2.36313	0.64318	0.27217
0.11	1.01211	0.01723	0.01702	0.61	1.38614	0.23673	0.17078	1.11	2.39100	0.65391	0.27349
0.12	1.01442	0.01964	0.01936	0.62	1.39939	0.24299	0.17364	1.12	2.41924	0.66475	0.27478
C.13	1.01693	0.02215	0.02178	0.63	1.41289	0.24933	0.17647	1.13	2.44785	0.67572	0.27604
0.14	1.01964	0.02476	0.02429	C.64	1.42663	0.25575	0.17927	1.14	2.47683	0.68679	0.27729
0.15	1.02255	0.02747	0.02687	0.65	1.44063	0.26224	0.18203	1.15	2.50619	0.69799	0.27851
C.16	1.02567	0.03028	0.02952	0.66	1.45488	0.26880	0.18476	1.16	2.53592	0.70930	0.27970
0.17	1.02898	0.03318	0.03224	0.67	1.46939	0.27545	0.18746	1.17	2.56604	0.72074	0.28087
C.18	1.03251	0.03617	0.03503	0.68	1.48415	0.28217	0.19012	1.18	2.59655	0.73229	0.28202
0.19	1.03623	0.03924	0.03787	0.69	1.49917	0.28897	0.19275	1.19	2.62744	0.74397	0.26315
0.20	1.04016	0.04241	0.04077	C.70	1.51445	0.29584	0.19535	1.20	2.65873	0.75577	0.28426
0.21	1.04429	0.04565	0.04372	0.71	1.52999	0.30280	0.19791	1.21	2.69042	0.76770	0.28534
0.22	1.04863	0.04898	0.04671	C.72	1.54580	0.30984	0.20044	1.22	2.72250	0.77975	0.28641
0.23	1.05318	0.05239	0.04975	0.73	1.56186	0.31695	0.20293	1.23	2.75499	0.79193	0.28745
0.24	1.05793	0.05588	0.05282	C.74	1.57820	0.32415	0.20539	1.24	2.78789	0.80424	0.28848
0.25	1.06289	0.05945	0.05594	0.75	1.59481	0.33143	0.20782	1.25	2.82120	0.81668	0.28948
0.26	1.06806	0.06310	0.05908	0.76	1.61168	0.33879	0.21021	1.26	2.85493	0.82925	0.29046
C.27	1.07343	0.06683	0.06226	0.77	1.62883	0.34623	0.21256	1.27	2.88908	0.84195	0.29143
C.28	1.07902	0.07063	0.06546	0.78	1.64626	0.35376	0.21488	1.28	2.92365	0.85479	0.29237
0.29	1.08481	0.07451	0.06868	C.79	1.66396	0.36137	0.21717	1.29	2.95865	0.86777	0.29330
C.30	1.09081	0.07846	0.07193	0.80	1.68194	0.36906	0.21942	1.30	2.99408	0.88088	0.29421
0.31	1.09703	0.08249	0.07519	0.81	1.70021	0.37684	0.22164	1.31	3.02995	0.89413	0.29510
C.32	1.10345	0.08659	0.07847	0.82	1.71875	0.38470	0.22383	1.32	3.06626	0.90752	0.29597
0.33	1.11009	0.09076	0.08176	0.83	1.73759	0.39266	0.22598	1.33	3.10301	0.92105	0.29683
0.34	1.11694	0.09501	0.08506	0.84	1.75671	0.40070	0.22809	1.34	3.14022	0.93473	0.29766
C.35	1.12401	0.09933	0.08837	0.85	1.77612	0.40882	0.23018	1.35	3.17787	0.94855	0.29849
C.36	1.13129	0.10372	0.09168	0.86	1.79582	0.41704	0.23223	1.36	3.21599	0.96252	0.29929
0.37	1.13878	0.10818	0.09500	C.87	1.81582	0.42535	0.23425	1.37	3.25457	0.97663	0.30008
0.38	1.14650	0.11272	0.09832	C.88	1.83612	0.43374	0.23623	1.38	3.29361	0.99090	0.30085
0.39	1.15443	0.11732	0.10163	0.89	1.85671	0.44223	0.23818	1.39	3.33313	1.00531	0.30161
0.40	1.16258	0.12200	0.10494	0.90	1.87761	0.45081	0.24010	1.40	3.37312	1.01988	0.30235
C.41	1.17094	0.12675	0.10825	0.91	1.89881	0.45949	0.24199	1.41	3.41360	1.03460	0.30308
0.42	1.17953	0.13157	0.11155	0.92	1.92032	0.46825	0.24384	1.42	3.45456	1.04948	0.30380
C.43	1.18834	0.13646	0.11483	C.93	1.94214	0.47712	0.24566	1.43	3.49601	1.06452	0.30449
0.44	1.19738	0.14142	0.11811	0.94	1.96427	0.48607	0.24746	1.44	3.53795	1.07971	0.30518
0.45	1.20663	0.14646	0.12138	C.95	1.98672	0.49513	0.24922	1.45	3.58040	1.09507	0.30585
C.46	1.21611	0.15156	0.12463	0.96	2.00949	0.50428	0.25095	1.46	3.62335	1.11059	0.30651
0.47	1.22582	0.15674	0.12786	0.97	2.03257	0.51353	0.25265	1.47	3.66681	1.12627	0.30715
0.48	1.23575	0.16198	0.13108	0.98	2.05598	0.52288	0.25432	1.48	3.71079	1.14212	0.30778
0.49	1.24592	0.16730	0.13428	0.99	2.07971	0.53233	0.25596	1.49	3.75529	1.15814	0.30840
0.50	1.25631	0.17269	0.13746	1.00	2.10378	0.54188	0.25757	1.50	3.80031	1.17433	0.30901

TABLE 4A. Lanchester-Clifford-Schläfli Functions $F_\alpha(x)$, $H_{1-\alpha}(x)$, and $T_\alpha(x)$ for $\alpha = 1/4$ and x from 0.00 to 1.50.

$\alpha = 1/4$

x	$F_{1/4}(x)$	$H_{3/4}(x)$	$T_{1/4}(x)$
1.50	3.80031	1.17433	0.30901
1.51	3.84587	1.19069	0.30960
1.52	3.89196	1.20722	0.31018
1.53	3.93859	1.22393	0.31075
1.54	3.98577	1.24081	0.31131
1.55	4.03350	1.25788	0.31186
1.56	4.08179	1.27512	0.31239
1.57	4.13065	1.29255	0.31292
1.58	4.18007	1.31017	0.31343
1.59	4.23007	1.32797	0.31394
1.60	4.28065	1.34596	0.31443
1.61	4.33182	1.36414	0.31491
1.62	4.38358	1.38251	0.31538
1.63	4.43594	1.40108	0.31585
1.64	4.48890	1.41985	0.31630
1.65	4.54248	1.43882	0.31675
1.66	4.59667	1.45798	0.31718
1.67	4.65149	1.47736	0.31761
1.68	4.70694	1.49694	0.31803
1.69	4.76302	1.51672	0.31844
1.70	4.81975	1.53672	0.31884
1.71	4.87713	1.55693	0.31923
1.72	4.93516	1.57735	0.31962
1.73	4.99386	1.59800	0.31999
1.74	5.05322	1.61886	0.32036
1.75	5.11327	1.63994	0.32072
1.76	5.17400	1.66125	0.32108
1.77	5.23541	1.68279	0.32142
1.78	5.29753	1.70455	0.32176
1.79	5.36035	1.72655	0.32210
1.80	5.42389	1.74878	0.32242
1.81	5.48814	1.77125	0.32274
1.82	5.55312	1.79396	0.32305
1.83	5.61884	1.81691	0.32336
1.84	5.68530	1.84010	0.32366
1.85	5.75251	1.86354	0.32395
1.86	5.82047	1.88724	0.32424
1.87	5.88921	1.91118	0.32452
1.88	5.95871	1.93538	0.32480
1.89	6.02900	1.95984	0.32507
1.90	6.10008	1.98456	0.32533
1.91	6.17196	2.00954	0.32559
1.92	6.24464	2.03479	0.32585
1.93	6.31814	2.06031	0.32610
1.94	6.39246	2.08611	0.32634
1.95	6.46761	2.11217	0.32658
1.96	6.54360	2.13852	0.32681
1.97	6.62044	2.16515	0.32704
1.98	6.69814	2.19206	0.32726
1.99	6.77671	2.21926	0.32748
2.00	6.85616	2.24675	0.32770

x	$F_{1/4}(x)$	$H_{3/4}(x)$	$T_{1/4}(x)$
2.0	6.85616	2.24675	0.32770
2.1	7.70085	2.53835	0.32962
2.2	8.64388	2.86267	0.33118
2.3	9.69609	3.22343	0.33245
2.4	10.86951	3.62473	0.33348
2.5	12.17756	4.07113	0.33431
2.6	13.63512	4.56771	0.33500
2.7	15.25875	5.12007	0.33555
2.8	17.06686	5.73448	0.33600
2.9	19.07988	6.41786	0.33637
3.0	21.32051	7.17792	0.33667
3.1	23.81395	8.02318	0.33691
3.2	26.58819	8.96313	0.33711
3.3	29.67429	10.00828	0.33727
3.4	33.10675	11.17032	0.33740
3.5	36.92382	12.46219	0.33751
3.6	41.16800	13.89826	0.33760
3.7	45.88641	15.49447	0.33767
3.8	51.13136	17.26852	0.33773
3.9	56.96088	19.24004	0.33778
4.0	63.43934	21.43078	0.33782
4.1	70.63817	23.86490	0.33785
4.2	78.63660	26.56919	0.33787
4.3	87.52252	29.57336	0.33789
4.4	97.39340	32.91036	0.33791
4.5	108.35735	36.61674	0.33793
4.6	120.53430	40.73304	0.33794
4.7	134.05721	45.30420	0.33795
4.8	149.07356	50.38008	0.33795
4.9	165.74688	56.01595	0.33796
5.0	184.25850	62.27311	0.33797
5.1	204.80949	69.21950	0.33797
5.2	227.62277	76.93050	0.33797
5.3	252.94553	85.48964	0.33798
5.4	281.05178	94.98954	0.33798
5.5	312.24535	105.53289	0.33798
5.6	346.86301	117.23353	0.33798
5.7	385.27813	130.21764	0.33798
5.8	427.90458	144.62511	0.33798
5.9	475.20112	160.61101	0.33799
6.0	527.67626	178.34720	0.33799

x	$F_{1/4}(x)$	$H_{3/4}(x)$	$T_{1/4}(x)$
6.0	527.67626	178.34720	0.33799
6.1	585.89365	198.02419	0.33799
6.2	650.47799	219.85313	0.33799
6.3	722.12168	244.06805	0.33799
6.4	801.59208	270.92829	0.33799
6.5	889.73967	300.72132	0.33799
6.6	987.50692	333.76568	0.33799
6.7	1095.93031	370.41439	0.33799
6.8	1216.19128	411.05867	0.33799
6.9	1349.54844	456.13200	0.33799
7.0	1497.43105	506.11478	0.33799
7.1	1661.41396	561.53928	0.33799
7.2	1843.24219	622.99529	0.33799
7.3	2044.84923	691.13633	0.33799
7.4	2268.37739	766.68645	0.33799
7.5	2516.20022	850.44792	0.33799
7.6	2790.94750	943.30954	0.33799
7.7	3095.53275	1046.25608	0.33799
7.8	3433.18381	1160.37849	0.33799
7.9	3807.47665	1286.88542	0.33799
8.0	4222.37283	1427.11584	0.33799
8.1	4682.26100	1582.55306	0.33799
8.2	5192.00279	1754.86026	0.33799
8.3	5756.98374	1945.79769	0.33799
8.4	6383.16955	2157.44170	0.33799
8.5	7077.16850	2392.00581	0.33799
8.6	7846.30050	2651.96407	0.33799
8.7	8698.67355	2940.05690	0.33799
8.8	9643.26842	3259.31970	0.33799
8.9	10690.03241	3613.11455	0.33799
9.0	11849.98315	4005.16529	0.33799
9.1	13135.32354	4439.59636	0.33799
9.2	14559.56902	4920.97585	0.33799
9.3	16137.68846	5454.36306	0.33799
9.4	17886.26020	6045.36129	0.33799
9.5	19823.64472	6700.17618	0.33799
9.6	21970.17593	7425.68038	0.33799
9.7	24348.37283	8229.48506	0.33799
9.8	26983.17397	9120.01918	0.33799
9.9	29902.19693	10106.61719	0.33799
10.0	33136.02562	11199.61611	0.33799

TABLE 4B. Lanchester-Clifford-Schläfli Functions $F_\alpha(x)$, $H_{1-\alpha}(x)$, and $T_\alpha(x)$ for $\alpha = 1/4$ and x from 1.50 to 10.0.

α = 3/4

x	$F_{3/4}(x)$	$H_{1/4}(x)$	$T_{3/4}(x)$	x	$F_{3/4}(x)$	$H_{1/4}(x)$	$T_{3/4}(x)$	x	$F_{3/4}(x)$	$H_{1/4}(x)$	$T_{3/4}(x)$
0.0	1.00000	0.0	0.0	0.50	1.08483	2.10140	1.93707	1.00	1.35788	3.42636	2.52332
0.01	1.00003	0.28285	0.28284	0.51	1.08832	2.12650	1.95393	1.01	1.36559	3.45621	2.53093
0.02	1.00013	0.40003	0.39998	0.52	1.09189	2.15158	1.97051	1.02	1.37340	3.48625	2.53841
0.03	1.00030	0.48999	0.48984	0.53	1.09553	2.17663	1.98683	1.03	1.38131	3.51650	2.54578
0.04	1.00053	0.56587	0.56556	0.54	1.09924	2.20166	2.00288	1.04	1.38932	3.54695	2.55302
0.05	1.00083	0.63277	0.63224	0.55	1.10303	2.22667	2.01868	1.05	1.39743	3.57761	2.56015
0.06	1.00120	0.69332	0.69249	0.56	1.10690	2.25169	2.03423	1.06	1.40564	3.60848	2.56716
0.07	1.00163	0.74907	0.74784	0.57	1.11084	2.27670	2.04953	1.07	1.41395	3.63957	2.57405
0.08	1.00213	0.80102	0.79932	0.58	1.11486	2.30172	2.06459	1.08	1.42236	3.67087	2.58083
0.09	1.00270	0.84990	0.84761	0.59	1.11895	2.32676	2.07942	1.09	1.43088	3.70240	2.58750
0.10	1.00334	0.89622	0.89324	0.60	1.12312	2.35182	2.09401	1.10	1.43950	3.73415	2.59407
0.11	1.00404	0.94035	0.93657	0.61	1.12737	2.37690	2.10837	1.11	1.44822	3.76613	2.60052
0.12	1.00480	0.98262	0.97792	0.62	1.13169	2.40202	2.12250	1.12	1.45705	3.79834	2.60687
0.13	1.00564	1.02325	1.01752	0.63	1.13610	2.42717	2.13641	1.13	1.46599	3.83079	2.61311
0.14	1.00654	1.06245	1.05555	0.64	1.14058	2.45237	2.15011	1.14	1.47503	3.86347	2.61925
0.15	1.00751	1.10038	1.09218	0.65	1.14514	2.47761	2.16359	1.15	1.48418	3.89640	2.62528
0.16	1.00855	1.13717	1.12753	0.66	1.14978	2.50291	2.17686	1.16	1.49344	3.92958	2.63122
0.17	1.00965	1.17294	1.16173	0.67	1.15450	2.52827	2.18993	1.17	1.50281	3.96300	2.63706
0.18	1.01083	1.20779	1.19486	0.68	1.15930	2.55369	2.20279	1.18	1.51229	3.99668	2.64280
0.19	1.01206	1.24180	1.22700	0.69	1.16418	2.57917	2.21545	1.19	1.52188	4.03061	2.64844
0.20	1.01337	1.27505	1.25823	0.70	1.16914	2.60474	2.22792	1.20	1.53158	4.06480	2.65400
0.21	1.01475	1.30761	1.28861	0.71	1.17418	2.63037	2.24019	1.21	1.54139	4.09926	2.65945
0.22	1.01619	1.33953	1.31819	0.72	1.17930	2.65609	2.25226	1.22	1.55132	4.13398	2.66482
0.23	1.01770	1.37086	1.34702	0.73	1.18450	2.68190	2.26415	1.23	1.56136	4.16897	2.67009
0.24	1.01928	1.40165	1.37514	0.74	1.18979	2.70780	2.27586	1.24	1.57152	4.20424	2.67528
0.25	1.02093	1.43195	1.40260	0.75	1.19516	2.73379	2.28738	1.25	1.58179	4.23979	2.68038
0.26	1.02264	1.46179	1.42943	0.76	1.20062	2.75989	2.29872	1.26	1.59218	4.27562	2.68539
0.27	1.02443	1.49121	1.45565	0.77	1.20615	2.78609	2.30989	1.27	1.60268	4.31173	2.69032
0.28	1.02628	1.52023	1.48130	0.78	1.21178	2.81239	2.32088	1.28	1.61331	4.34813	2.69516
0.29	1.02820	1.54889	1.50641	0.79	1.21748	2.83881	2.33170	1.29	1.62405	4.38482	2.69992
0.30	1.03019	1.57722	1.53099	0.80	1.22328	2.86534	2.34235	1.30	1.63492	4.42181	2.70460
0.31	1.03225	1.60523	1.55507	0.81	1.22916	2.89200	2.35283	1.31	1.64590	4.45909	2.70921
0.32	1.03438	1.63295	1.57867	0.82	1.23512	2.91878	2.36315	1.32	1.65701	4.49669	2.71373
0.33	1.03658	1.66041	1.60181	0.83	1.24117	2.94568	2.37331	1.33	1.66825	4.53458	2.71817
0.34	1.03885	1.68762	1.62450	0.84	1.24731	2.97272	2.38331	1.34	1.67960	4.57279	2.72254
0.35	1.04119	1.71460	1.64676	0.85	1.25354	2.99990	2.39315	1.35	1.69109	4.61131	2.72683
0.36	1.04360	1.74136	1.66861	0.86	1.25985	3.02721	2.40283	1.36	1.70270	4.65015	2.73105
0.37	1.04608	1.76793	1.69005	0.87	1.26626	3.05467	2.41236	1.37	1.71443	4.68931	2.73520
0.38	1.04863	1.79432	1.71110	0.88	1.27275	3.08228	2.42174	1.38	1.72630	4.72880	2.73927
0.39	1.05125	1.82054	1.73178	0.89	1.27934	3.11003	2.43098	1.39	1.73829	4.76861	2.74328
0.40	1.05395	1.84661	1.75209	0.90	1.28601	3.13795	2.44006	1.40	1.75041	4.80876	2.74721
0.41	1.05671	1.87254	1.77204	0.91	1.29278	3.16601	2.44900	1.41	1.76267	4.84925	2.75108
0.42	1.05954	1.89834	1.79165	0.92	1.29963	3.19425	2.45780	1.42	1.77506	4.89007	2.75488
0.43	1.06245	1.92402	1.81092	0.93	1.30658	3.22264	2.46646	1.43	1.78758	4.93125	2.75862
0.44	1.06543	1.94960	1.82987	0.94	1.31363	3.25121	2.47499	1.44	1.80024	4.97277	2.76228
0.45	1.06848	1.97508	1.84849	0.95	1.32076	3.27995	2.48337	1.45	1.81303	5.01464	2.76589
0.46	1.07161	2.00048	1.86680	0.96	1.32800	3.30886	2.49162	1.46	1.82596	5.05687	2.76943
0.47	1.07480	2.02580	1.88481	0.97	1.33532	3.33796	2.49974	1.47	1.83903	5.09945	2.77291
0.48	1.07807	2.05105	1.90252	0.98	1.34274	3.36724	2.50773	1.48	1.85223	5.14241	2.77633
0.49	1.08142	2.07625	1.91994	0.99	1.35026	3.39670	2.51559	1.49	1.86558	5.18573	2.77969
0.50	1.08483	2.10140	1.93707	1.00	1.35788	3.42636	2.52332	1.50	1.87907	5.22942	2.78299

TABLE 5A. Lanchester-Clifford-Schläfli Functions $F_\alpha(x)$, $H_{1-\alpha}(x)$, and $T_\alpha(x)$ for $\alpha = 3/4$ and x from 0.00 to 1.50.

$\alpha = 3/4$

x	$F_{3/4}(x)$	$H_{1/4}(x)$	$T_{3/4}(x)$
1.50	1.87907	5.22942	2.78299
1.51	1.89270	5.27350	2.78623
1.52	1.90648	5.31795	2.78941
1.53	1.92040	5.36279	2.79254
1.54	1.93446	5.40801	2.79561
1.55	1.94868	5.45363	2.79863
1.56	1.96304	5.49965	2.80160
1.57	1.97755	5.54607	2.80451
1.58	1.99222	5.59289	2.80737
1.59	2.00704	5.64013	2.81018
1.60	2.02201	5.68778	2.81294
1.61	2.03713	5.73585	2.81565
1.62	2.05242	5.78434	2.81831
1.63	2.06786	5.83326	2.82092
1.64	2.08346	5.88261	2.82349
1.65	2.09922	5.93241	2.82601
1.66	2.11514	5.98264	2.82849
1.67	2.13122	6.03332	2.83092
1.68	2.14747	6.08445	2.83331
1.69	2.16389	6.13603	2.83565
1.70	2.18047	6.18808	2.83795
1.71	2.19723	6.24059	2.84021
1.72	2.21415	6.29357	2.84243
1.73	2.23124	6.34703	2.84461
1.74	2.24851	6.40096	2.84675
1.75	2.26596	6.45538	2.84885
1.76	2.28358	6.51029	2.85092
1.77	2.30138	6.56569	2.85294
1.78	2.31935	6.62159	2.85493
1.79	2.33751	6.67800	2.85688
1.80	2.35585	6.73492	2.85880
1.81	2.37438	6.79235	2.86068
1.82	2.39309	6.85030	2.86253
1.83	2.41199	6.90877	2.86434
1.84	2.43108	6.96778	2.86612
1.85	2.45036	7.02732	2.86787
1.86	2.46983	7.08741	2.86959
1.87	2.48950	7.14804	2.87127
1.88	2.50937	7.20922	2.87292
1.89	2.52943	7.27096	2.87455
1.90	2.54969	7.33326	2.87614
1.91	2.57015	7.39613	2.87770
1.92	2.59081	7.45957	2.87924
1.93	2.61168	7.52360	2.88075
1.94	2.63276	7.58821	2.88222
1.95	2.65405	7.65341	2.88368
1.96	2.67554	7.71921	2.88510
1.97	2.69725	7.78562	2.88650
1.98	2.71918	7.85263	2.88787
1.99	2.74132	7.92025	2.88922
2.00	2.76367	7.98850	2.89054

x	$F_{3/4}(x)$	$H_{1/4}(x)$	$T_{3/4}(x)$
2.0	2.76367	7.98850	2.89054
2.1	2.99970	8.70654	2.90247
2.2	3.25984	9.49380	2.91235
2.3	3.54639	10.35732	2.92052
2.4	3.86190	11.30482	2.92727
2.5	4.20915	12.34477	2.93284
2.6	4.59124	13.48645	2.93743
2.7	5.01157	14.74009	2.94121
2.8	5.47389	16.11690	2.94433
2.9	5.98232	17.62923	2.94689
3.0	6.54143	19.29064	2.94900
3.1	7.15622	21.11607	2.95073
3.2	7.83223	23.12194	2.95215
3.3	8.57555	25.32635	2.95332
3.4	9.39287	27.74918	2.95428
3.5	10.29158	30.41235	2.95507
3.6	11.27981	33.33997	2.95572
3.7	12.36654	36.55859	2.95625
3.8	13.56162	40.09746	2.95669
3.9	14.87592	43.98876	2.95705
4.0	16.32142	48.26796	2.95734
4.1	17.911[illegible]	52.97409	2.95758
4.2	19.66009	58.15016	2.95778
4.3	21.58379	63.84353	2.95794
4.4	23.70002	70.10637	2.95807
4.5	26.02821	76.99614	2.95818
4.6	28.58973	84.57615	2.95827
4.7	31.40816	92.91611	2.95834
4.8	34.50945	102.09286	2.95840
4.9	37.92220	112.19102	2.95845
5.0	41.67792	123.30382	2.95849
5.1	45.81135	135.53399	2.95853
5.2	50.36069	148.99473	2.95855
5.3	55.36813	163.81073	2.95857
5.4	60.88010	180.11942	2.95859
5.5	66.94778	198.07221	2.95861
5.6	73.62756	217.83597	2.95862
5.7	80.98160	239.59457	2.95863
5.8	89.07837	263.55064	2.95864
5.9	97.99335	289.92750	2.95864
6.0	107.80972	318.97126	2.95865

x	$F_{3/4}(x)$	$H_{1/4}(x)$	$T_{3/4}(x)$
6.0	107.80972	318.97126	2.95865
6.1	118.61917	350.95316	2.95865
6.2	130.52272	386.17215	2.95866
6.3	143.63175	424.95772	2.95866
6.4	158.06900	467.67305	2.95866
6.5	173.96977	514.71843	2.95867
6.6	191.48319	566.53511	2.95867
6.7	210.77365	623.60946	2.95867
6.8	232.02236	686.47762	2.95867
6.9	255.42908	755.73062	2.95867
7.0	281.21402	832.01997	2.95867
7.1	309.61993	916.06393	2.95867
7.2	340.91444	1008.65430	2.95867
7.3	375.39259	1110.66402	2.95867
7.4	413.37966	1223.05550	2.95867
7.5	455.23428	1346.88978	2.95867
7.6	501.35184	1483.33670	2.95867
7.7	552.16828	1633.68608	2.95867
7.8	608.16425	1799.36002	2.95867
7.9	669.86974	1981.92656	2.95867
8.0	737.86911	2183.11464	2.95867
8.1	812.80670	2404.83066	2.95867
8.2	895.39301	2649.17676	2.95867
8.3	986.41151	2918.47093	2.95867
8.4	1086.72611	3215.26927	2.95867
8.5	1197.28949	3542.39043	2.95867
8.6	1319.15223	3902.94271	2.95867
8.7	1453.47287	4300.35385	2.95868
8.8	1601.52902	4738.40392	2.95868
8.9	1764.72967	5221.26163	2.95868
9.0	1944.62868	5753.52437	2.95868
9.1	2142.93973	6340.26235	2.95868
9.2	2361.55278	6987.06735	2.95868
9.3	2602.55224	7700.10648	2.95868
9.4	2868.23703	8486.18146	2.95868
9.5	3161.14265	9352.79405	2.95868
9.6	3484.06564	10308.21826	2.95868
9.7	3840.09044	11361.57997	2.95868
9.8	4232.61910	12522.94477	2.95868
9.9	4665.40409	13803.41494	2.95868
10.0	5142.58436	15215.23636	2.95868

TABLE 5B. Lanchester-Clifford-Schläfli Functions $F_\alpha(x)$, $H_{1-\alpha}(x)$, and $T_\alpha(x)$ for $\alpha = 3/4$ and x from 1.50 to 10.0.

$\alpha = 1/5$

x	$F_{1/5}(x)$	$H_{4/5}(x)$	$T_{1/5}(x)$	x	$F_{1/5}(x)$	$H_{4/5}(x)$	$T_{1/5}(x)$	x	$F_{1/5}(x)$	$H_{4/5}(x)$	$T_{1/5}(x)$
0.0	1.00000	0.0	0.0	0.50	1.32072	0.14080	0.10661	1.00	2.38524	0.47223	0.19798
0.01	1.00013	0.00026	0.00026	0.51	1.33402	0.14553	0.10909	1.01	2.41596	0.48109	0.19913
0.02	1.00050	0.00079	0.00079	0.52	1.34762	0.15034	0.11156	1.02	2.44711	0.49005	0.20025
0.03	1.00113	0.00151	0.00151	0.53	1.36151	0.15521	0.11400	1.03	2.47869	0.49911	0.20136
0.04	1.00200	0.00239	0.00239	0.54	1.37569	0.16016	0.11642	1.04	2.51070	0.50827	0.20244
0.05	1.00313	0.00342	0.00341	0.55	1.39018	0.16518	0.11882	1.05	2.54315	0.51753	0.20350
0.06	1.00450	0.00458	0.00456	0.56	1.40496	0.17027	0.12119	1.06	2.57604	0.52691	0.20454
0.C7	1.00613	0.00586	0.00582	0.57	1.42004	0.17543	0.12354	1.07	2.60937	0.53638	0.20556
0.08	1.00801	0.00725	0.00720	0.58	1.43542	0.18066	0.12586	1.08	2.64315	0.54597	0.20656
C.09	1.01013	0.00876	0.00867	0.59	1.45111	0.18597	0.12816	1.09	2.67739	0.55566	0.20754
0.10	1.01251	0.01037	0.01024	0.60	1.46711	0.19135	0.13043	1.10	2.71209	0.56547	0.20850
0.11	1.01514	0.01208	0.01190	0.61	1.48341	0.19680	0.13267	1.11	2.74724	0.57538	0.20944
0.12	1.01803	0.01389	0.01365	0.62	1.50002	0.20233	C.13488	1.12	2.78286	0.58540	0.21036
0.13	1.02116	0.01580	0.01547	0.63	1.51695	0.20793	0.13707	1.13	2.81895	0.59554	0.21126
C.14	1.02455	0.01779	0.01737	0.64	1.53419	0.21360	0.13923	1.14	2.85552	0.60579	0.21215
0.15	1.02819	0.01988	0.01933	C.65	1.55174	0.21935	0.14136	1.15	2.89257	0.61616	0.21301
0.16	1.03209	0.02205	0.02136	0.66	1.56962	0.22518	0.14346	1.16	2.93010	0.62664	0.21386
0.17	1.03623	0.02431	0.02346	0.67	1.58781	0.23108	0.14553	1.17	2.96812	0.63723	0.21469
0.18	1.04064	0.02665	0.02561	C.68	1.60633	0.23706	C.14758	1.18	3.00663	0.64795	0.21551
0.19	1.04529	0.02907	0.02781	0.69	1.62518	0.24311	0.14959	1.19	3.04564	0.65879	0.21631
0.20	1.05021	0.03157	0.03006	0.70	1.64435	0.24924	0.15158	1.20	3.08515	0.66975	0.21709
0.21	1.05538	0.03416	0.03236	0.71	1.66385	0.25545	0.15353	1.21	3.12517	0.68082	0.21785
0.22	1.06081	0.03682	0.03471	0.72	1.68369	0.26174	0.15546	1.22	3.16570	0.69203	0.21860
0.23	1.06649	0.03956	0.03709	0.73	1.70386	0.26811	0.15735	1.23	3.20675	0.70335	0.21934
0.24	1.07243	0.04237	0.03951	0.74	1.72436	0.27455	0.15922	1.24	3.24832	0.71481	0.22005
0.25	1.07863	0.04526	0.04196	0.75	1.74521	0.28108	0.16106	1.25	3.29042	0.72639	C.22076
0.26	1.08510	0.04823	0.04444	C.76	1.76640	0.28769	0.16287	1.26	3.33305	0.73810	0.22145
0.27	1.09182	0.05127	0.04696	C.77	1.78794	0.29437	0.16464	1.27	3.37621	0.74994	0.22212
0.28	1.09880	0.05438	0.04949	0.78	1.80982	0.30114	0.16639	1.28	3.41992	0.76191	0.22278
0.29	1.10605	0.05757	0.05205	0.79	1.83205	0.30800	0.16811	1.29	3.46418	0.77401	0.22343
0.30	1.11356	0.06082	0.05462	0.80	1.85464	0.31493	0.16981	1.30	3.50900	0.78624	0.22407
0.31	1.12133	0.06415	0.05721	0.81	1.87759	0.32195	0.17147	1.31	3.55437	0.79862	0.22469
0.32	1.12937	0.06756	0.05982	0.82	1.90089	0.32905	0.17311	1.32	3.60030	0.81113	0.22529
0.33	1.13768	0.07103	0.06243	0.83	1.92455	0.33624	0.17471	1.33	3.64681	0.82377	0.22589
C.34	1.14625	0.07457	0.06506	0.84	1.94858	0.34352	0.17629	1.34	3.69389	0.83656	0.22647
0.35	1.15509	0.07819	0.06769	0.85	1.97298	0.35088	0.17784	1.35	3.74156	0.84949	0.22704
0.36	1.16420	0.08187	0.07033	0.86	1.99775	0.35833	0.17937	1.36	3.78981	0.86256	0.22760
0.37	1.17358	0.08563	0.07296	0.87	2.02289	0.36587	0.18086	1.37	3.83865	0.87577	0.22815
0.38	1.18323	0.08945	0.07560	C.88	2.04841	0.37349	0.18233	1.38	3.88809	0.88913	0.22868
0.39	1.19315	0.09335	0.07824	0.89	2.07431	0.38121	0.18378	1.39	3.93814	0.90264	0.22921
0.40	1.20335	0.09731	0.08087	0.90	2.10059	0.38901	0.18519	1.40	3.98880	0.91630	0.22972
0.41	1.21383	0.10135	0.08349	0.91	2.12726	0.39691	0.18658	1.41	4.04008	0.93011	0.23022
0.42	1.22458	0.10545	0.08611	0.92	2.15432	0.40490	0.18795	1.42	4.09198	0.94407	0.23071
0.43	1.23561	0.10963	0.08872	0.93	2.18177	0.41298	0.18929	1.43	4.14451	0.95818	0.23119
0.44	1.24692	0.11387	0.09132	0.94	2.20962	0.42116	0.19060	1.44	4.19767	0.97245	0.23166
0.45	1.25851	0.11818	0.09391	0.55	2.23787	0.42943	0.19189	1.45	4.25148	0.98688	0.23212
0.46	1.27038	0.12257	0.09648	C.96	2.26652	0.43779	0.19316	1.46	4.30594	1.00146	0.23258
0.47	1.28253	0.12702	0.09904	0.97	2.29558	0.44626	0.19440	1.47	4.36105	1.01620	0.23302
0.48	1.29497	0.13154	0.10158	0.98	2.32505	0.45482	0.19562	1.48	4.41683	1.03111	0.23345
0.49	1.30770	0.13614	0.10410	0.99	2.35494	0.46347	0.19681	1.49	4.47327	1.04618	0.23387
C.50	1.32072	0.14080	0.10661	1.00	2.38524	0.47223	0.19798	1.50	4.53040	1.06142	0.23429

TABLE 6A. Lanchester-Clifford-Schläfli Functions $F_\alpha(x)$, $H_{1-\alpha}(x)$, and $T_\alpha(x)$ for $\alpha = 1/5$ and x from 0.00 to 1.50.

$\alpha = 1/5$

x	$F_{1/5}(x)$	$H_{4/5}(x)$	$T_{1/5}(x)$
1.50	4.53040	1.06142	0.23429
1.51	4.58820	1.07682	0.23469
1.52	4.64669	1.09240	0.23509
1.53	4.70588	1.10814	0.23548
1.54	4.76578	1.12406	0.23586
1.55	4.82639	1.14015	0.23623
1.56	4.88771	1.15642	0.23660
1.57	4.94976	1.17286	0.23695
1.58	5.01255	1.18949	0.23730
1.59	5.07607	1.20630	0.23764
1.60	5.14035	1.22330	0.23798
1.61	5.20538	1.24048	0.23831
1.62	5.27117	1.25785	0.23863
1.63	5.33774	1.27541	0.23894
1.64	5.40509	1.29316	0.23925
1.65	5.47322	1.31111	0.23955
1.66	5.54216	1.32925	0.23984
1.67	5.61189	1.34759	0.24013
1.68	5.68244	1.36614	0.24041
1.69	5.75382	1.38488	0.24069
1.70	5.82602	1.40383	0.24096
1.71	5.89906	1.42299	0.24122
1.72	5.97295	1.44236	0.24148
1.73	6.04770	1.46195	0.24174
1.74	6.12331	1.48174	0.24198
1.75	6.19979	1.50175	0.24223
1.76	6.27716	1.52199	0.24246
1.77	6.35542	1.54244	0.24270
1.78	6.43459	1.56312	0.24292
1.79	6.51467	1.58402	0.24315
1.80	6.59566	1.60516	0.24337
1.81	6.67759	1.62652	0.24358
1.82	6.76046	1.64812	0.24379
1.83	6.84428	1.66995	0.24399
1.84	6.92907	1.69203	0.24419
1.85	7.01482	1.71434	0.24439
1.86	7.10155	1.73690	0.24458
1.87	7.18928	1.75971	0.24477
1.88	7.27801	1.78276	0.24495
1.89	7.36775	1.80607	0.24513
1.90	7.45851	1.82963	0.24531
1.91	7.55031	1.85346	0.24548
1.92	7.64315	1.87754	0.24565
1.93	7.73705	1.90188	0.24581
1.94	7.83201	1.92649	0.24598
1.95	7.92806	1.95137	0.24614
1.96	8.02519	1.97653	0.24629
1.97	8.12343	2.00195	0.24644
1.98	8.22278	2.02766	0.24659
1.99	8.32325	2.05365	0.24674
2.00	8.42486	2.07992	0.24688

x	$F_{1/5}(x)$	$H_{4/5}(x)$	$T_{1/5}(x)$
2.0	8.42486	2.07992	0.24688
2.1	9.50623	2.35895	0.24815
2.2	10.71544	2.67002	0.24917
2.3	12.06679	3.01677	0.25001
2.4	13.57620	3.40328	0.25068
2.5	15.26139	3.83405	0.25123
2.6	17.14209	4.31413	0.25167
2.7	19.24023	4.84908	0.25203
2.8	21.58022	5.44513	0.25232
2.9	24.18919	6.10916	0.25256
3.0	27.09729	6.84885	0.25275
3.1	30.33805	7.67272	0.25291
3.2	33.94870	8.59022	0.25304
3.3	37.97065	9.61186	0.25314
3.4	42.44987	10.74932	0.25322
3.5	47.43742	12.01556	0.25329
3.6	52.99005	13.42498	0.25335
3.7	59.17075	14.99356	0.25339
3.8	66.04948	16.73907	0.25343
3.9	73.70395	18.68120	0.25346
4.0	82.22040	20.84185	0.25349
4.1	91.69462	23.24531	0.25351
4.2	102.23292	25.91854	0.25352
4.3	113.95332	28.89149	0.25354
4.4	126.98683	32.19738	0.25355
4.5	141.47886	35.87310	0.25356
4.6	157.59081	39.95957	0.25357
4.7	175.50179	44.50223	0.25357
4.8	195.41055	49.55148	0.25358
4.9	217.53767	55.16326	0.25358
5.0	242.12789	61.39965	0.25358
5.1	269.45279	68.32952	0.25359
5.2	299.81367	76.02927	0.25359
5.3	333.54482	84.58370	0.25359
5.4	371.01708	94.08684	0.25359
5.5	412.64189	104.64303	0.25359
5.6	458.87562	116.36803	0.25359
5.7	510.22452	129.39020	0.25359
5.8	567.25012	143.85195	0.25360
5.9	630.57526	159.91124	0.25360
6.0	700.89071	177.74325	0.25360

x	$F_{1/5}(x)$	$H_{4/5}(x)$	$T_{1/5}(x)$
6.0	700.89071	177.74325	0.25360
6.1	778.96261	197.54227	0.25360
6.2	865.64062	219.52377	0.25360
6.3	961.86701	243.92671	0.25360
6.4	1068.68667	271.01608	0.25360
6.5	1187.25832	301.08572	0.25360
6.6	1318.86678	334.46147	0.25360
6.7	1464.93670	371.50461	0.25360
6.8	1627.04773	412.61574	0.25360
6.9	1806.95125	458.23901	0.25360
7.0	2006.58906	508.86685	0.25360
7.1	2228.11398	565.04522	0.25360
7.2	2473.91267	627.37937	0.25360
7.3	2746.63101	696.54029	0.25360
7.4	3049.20213	773.27182	0.25360
7.5	3384.87750	858.39852	0.25360
7.6	3757.26135	952.83443	0.25360
7.7	4170.34888	1057.59271	0.25360
7.8	4628.56851	1173.79641	0.25360
7.9	5136.82869	1302.69030	0.25360
8.0	5700.56986	1445.65407	0.25360
8.1	6325.82188	1604.21688	0.25360
8.2	7019.26772	1780.07351	0.25360
8.3	7786.31405	1975.10229	0.25360
8.4	8641.16934	2191.38488	0.25360
8.5	9586.93047	2431.22821	0.25360
8.6	10635.67862	2697.18887	0.25360
8.7	11798.58552	2992.09998	0.25360
8.8	13088.03108	3319.10107	0.25360
8.9	14517.73379	3681.67111	0.25360
9.0	16102.89504	4083.66516	0.25360
9.1	17860.35910	4529.35490	0.25360
9.2	19808.79019	5023.47352	0.25360
9.3	21968.86871	5571.26556	0.25360
9.4	24363.50854	6178.54192	0.25360
9.5	27018.09769	6851.74096	0.25360
9.6	29960.76485	7597.99607	0.25360
9.7	33222.67468	8425.21055	0.25360
9.8	36838.35470	9342.14050	0.25360
9.9	40846.05746	10358.48672	0.25360
10.0	45288.16157	11484.99634	0.25360

TABLE 6B. Lanchester-Clifford-Schläfli Functions $F_\alpha(x)$, $H_{1-\alpha}(x)$, and $T_\alpha(x)$ for $\alpha = 1/5$ and x from 1.50 to 10.0.

α = 2/5

x	$F_{2/5}(x)$	$H_{3/5}(x)$	$T_{2/5}(x)$	x	$F_{2/5}(x)$	$H_{3/5}(x)$	$T_{2/5}(x)$	x	$F_{2/5}(x)$	$H_{3/5}(x)$	$T_{2/5}(x)$
0.0	1.00000	0.0	0.0	0.50	1.15977	0.32826	0.28304	1.00	1.68278	0.84439	0.50178
0.01	1.00006	0.00289	0.00289	0.51	1.16637	0.33668	0.28865	1.01	1.69773	0.85707	0.50483
0.02	1.00025	0.00664	0.00663	0.52	1.17312	0.34516	0.29422	1.02	1.71288	0.86986	0.50784
0.03	1.00056	0.01079	0.01079	0.53	1.18001	0.35371	0.29975	1.03	1.72823	0.88276	0.51079
0.04	1.00100	0.01525	0.01523	0.54	1.18704	0.36233	0.30524	1.04	1.74378	0.89578	0.51370
0.05	1.00156	0.01993	0.01990	0.55	1.19422	0.37102	0.31068	1.05	1.75954	0.90891	0.51656
0.06	1.00225	0.02481	0.02475	0.56	1.20155	0.37977	0.31607	1.06	1.77551	0.92216	0.51938
0.07	1.00306	0.02986	0.02977	0.57	1.20902	0.38860	0.32141	1.07	1.79168	0.93553	0.52215
0.08	1.00400	0.03506	0.03492	0.58	1.21664	0.39749	0.32671	1.08	1.80806	0.94901	0.52488
0.09	1.00507	0.04039	0.04018	0.59	1.22441	0.40646	0.33196	1.09	1.82466	0.96261	0.52756
0.10	1.00626	0.04584	0.04556	0.60	1.23232	0.41549	0.33716	1.10	1.84146	0.97634	0.53020
0.11	1.00757	0.05142	0.05103	0.61	1.24039	0.42460	0.34231	1.11	1.85848	0.99019	0.53279
0.12	1.00901	0.05710	0.05659	0.62	1.24861	0.43378	0.34741	1.12	1.87572	1.00416	0.53535
0.13	1.01058	0.06288	0.06222	0.63	1.25698	0.44304	0.35246	1.13	1.89318	1.01826	0.53786
0.14	1.01227	0.06875	0.06792	0.64	1.26550	0.45237	0.35746	1.14	1.91086	1.03248	0.54032
0.15	1.01409	0.07472	0.07368	0.65	1.27417	0.46177	0.36241	1.15	1.92876	1.04683	0.54275
0.16	1.01604	0.08078	0.07950	0.66	1.28300	0.47125	0.36730	1.16	1.94688	1.06131	0.54514
0.17	1.01811	0.08692	0.08537	0.67	1.29198	0.48080	0.37214	1.17	1.96523	1.07593	0.54748
0.18	1.02031	0.09314	0.09129	0.68	1.30112	0.49043	0.37693	1.18	1.98381	1.09067	0.54979
0.19	1.02264	0.09944	0.09724	0.69	1.31042	0.50014	0.38166	1.19	2.00262	1.10555	0.55206
0.20	1.02509	0.10582	0.10323	0.70	1.31988	0.50992	0.38634	1.20	2.02166	1.12057	0.55428
0.21	1.02767	0.11227	0.10925	0.71	1.32949	0.51979	0.39097	1.21	2.04093	1.13572	0.55647
0.22	1.03038	0.11880	0.11529	0.72	1.33927	0.52973	0.39554	1.22	2.06044	1.15102	0.55863
0.23	1.03322	0.12539	0.12136	0.73	1.34921	0.53976	0.40006	1.23	2.08019	1.16645	0.56074
0.24	1.03619	0.13206	0.12745	0.74	1.35930	0.54987	0.40452	1.24	2.10018	1.18202	0.56282
0.25	1.03928	0.13880	0.13355	0.75	1.36957	0.56006	0.40893	1.25	2.12041	1.19774	0.56486
0.26	1.04251	0.14560	0.13966	0.76	1.37999	0.57033	0.41328	1.26	2.14089	1.21360	0.56687
0.27	1.04586	0.15247	0.14578	0.77	1.39059	0.58069	0.41758	1.27	2.16161	1.22961	0.56884
0.28	1.04934	0.15941	0.15191	0.78	1.40135	0.59113	0.42183	1.28	2.18258	1.24577	0.57078
0.29	1.05296	0.16641	0.15804	0.79	1.41227	0.60166	0.42602	1.29	2.20380	1.26208	0.57268
0.30	1.05670	0.17348	0.16417	0.80	1.42337	0.61227	0.43016	1.30	2.22528	1.27854	0.57455
0.31	1.06058	0.18061	0.17030	0.81	1.43464	0.62298	0.43424	1.31	2.24701	1.29515	0.57639
0.32	1.06459	0.18781	0.17642	0.82	1.44608	0.63377	0.43827	1.32	2.26900	1.31192	0.57819
0.33	1.06873	0.19507	0.18253	0.83	1.45769	0.64465	0.44224	1.33	2.29125	1.32884	0.57996
0.34	1.07300	0.20240	0.18863	0.84	1.46947	0.65562	0.44616	1.34	2.31377	1.34592	0.58170
0.35	1.07740	0.20979	0.19471	0.85	1.48143	0.66669	0.45003	1.35	2.33654	1.36316	0.58341
0.36	1.08194	0.21724	0.20079	0.86	1.49357	0.67784	0.45384	1.36	2.35959	1.38057	0.58509
0.37	1.08661	0.22475	0.20684	0.87	1.50588	0.68909	0.45760	1.37	2.38291	1.39814	0.58674
0.38	1.09142	0.23233	0.21287	0.88	1.51838	0.70044	0.46131	1.38	2.40650	1.41587	0.58835
0.39	1.09636	0.23997	0.21888	0.89	1.53105	0.71188	0.46496	1.39	2.43036	1.43377	0.58994
0.40	1.10144	0.24768	0.22487	0.90	1.54391	0.72342	0.46856	1.40	2.45450	1.45184	0.59150
0.41	1.10665	0.25545	0.23083	0.91	1.55695	0.73506	0.47211	1.41	2.47892	1.47009	0.59303
0.42	1.11200	0.26328	0.23676	0.92	1.57017	0.74679	0.47561	1.42	2.50363	1.48850	0.59454
0.43	1.11748	0.27118	0.24267	0.93	1.58358	0.75863	0.47906	1.43	2.52862	1.50709	0.59601
0.44	1.12311	0.27914	0.24854	0.94	1.59718	0.77057	0.48246	1.44	2.55390	1.52586	0.59746
0.45	1.12887	0.28716	0.25438	0.95	1.61096	0.78261	0.48580	1.45	2.57946	1.54480	0.59889
0.46	1.13477	0.29525	0.26019	0.96	1.62494	0.79475	0.48910	1.46	2.60533	1.56393	0.60028
0.47	1.14081	0.30340	0.26596	0.97	1.63911	0.80700	0.49234	1.47	2.63148	1.58324	0.60165
0.48	1.14699	0.31162	0.27169	0.98	1.65347	0.81936	0.49554	1.48	2.65794	1.60273	0.60300
0.49	1.15331	0.31991	0.27738	0.99	1.66802	0.83182	0.49868	1.49	2.68470	1.62241	0.60432
0.50	1.15977	0.32826	0.28304	1.00	1.68278	0.84439	0.50178	1.50	2.71176	1.64228	0.60561

TABLE 7A. Lanchester-Clifford-Schläfli Functions $F_\alpha(x)$, $H_{1-\alpha}(x)$, and $T_\alpha(x)$ for $\alpha = 2/5$ and x from 0.00 to 1.50.

$\alpha = 2/5$

x	$F_{2/5}(x)$	$H_{3/5}(x)$	$T_{2/5}(x)$
1.50	2.71176	1.64228	0.60561
1.51	2.73913	1.66234	0.60689
1.52	2.76681	1.68259	0.60813
1.53	2.79481	1.70304	0.60936
1.54	2.82312	1.72369	0.61056
1.55	2.85175	1.74453	0.61174
1.56	2.88070	1.76558	0.61290
1.57	2.90998	1.78683	0.61403
1.58	2.93959	1.80828	0.61515
1.59	2.96952	1.82995	0.61624
1.60	2.99980	1.85182	0.61731
1.61	3.03041	1.87391	0.61837
1.62	3.06136	1.89621	0.61940
1.63	3.09266	1.91873	0.62041
1.64	3.12430	1.94147	0.62141
1.65	3.15630	1.96443	0.62238
1.66	3.18865	1.98761	0.62334
1.67	3.22136	2.01102	0.62428
1.68	3.25443	2.03466	0.62520
1.69	3.28786	2.05854	0.62610
1.70	3.32166	2.08264	0.62699
1.71	3.35584	2.10699	0.62786
1.72	3.39039	2.13157	0.62871
1.73	3.42532	2.15640	0.62955
1.74	3.46064	2.18146	0.63037
1.75	3.49634	2.20678	0.63117
1.76	3.53243	2.23235	0.63196
1.77	3.56892	2.25817	0.63273
1.78	3.60580	2.28424	0.63349
1.79	3.64309	2.31057	0.63423
1.80	3.68078	2.33716	0.63496
1.81	3.71888	2.36402	0.63568
1.82	3.75740	2.39114	0.63638
1.83	3.79634	2.41853	0.63707
1.84	3.83569	2.44620	0.63775
1.85	3.87548	2.47413	0.63841
1.86	3.91569	2.50235	0.63906
1.87	3.95634	2.53085	0.63969
1.88	3.99743	2.55963	0.64032
1.89	4.03897	2.58869	0.64093
1.90	4.08095	2.61805	0.64153
1.91	4.12338	2.64770	0.64212
1.92	4.16627	2.67764	0.64270
1.93	4.20963	2.70789	0.64326
1.94	4.25345	2.73844	0.64382
1.95	4.29774	2.76929	0.64436
1.96	4.34250	2.80045	0.64489
1.97	4.38775	2.83192	0.64542
1.98	4.43348	2.86371	0.64593
1.99	4.47970	2.89582	0.64643
2.00	4.52641	2.92825	0.64692

x	$F_{2/5}(x)$	$H_{3/5}(x)$	$T_{2/5}(x)$
2.0	4.52641	2.92825	0.64692
2.1	5.02188	3.27101	0.65135
2.2	5.57258	3.64995	0.65498
2.3	6.18434	4.06905	0.65796
2.4	6.86361	4.53271	0.66040
2.5	7.61753	5.04580	0.66239
2.6	8.45406	5.61371	0.66403
2.7	9.38197	6.24240	0.66536
2.8	10.41103	6.93848	0.66645
2.9	11.55204	7.70923	0.66735
3.0	12.81695	8.56274	0.66808
3.1	14.21903	9.50795	0.66868
3.2	15.77295	10.55473	0.66917
3.3	17.49496	11.71404	0.66957
3.4	19.40306	12.99799	0.66989
3.5	21.51718	14.41998	0.67016
3.6	23.85938	15.99486	0.67038
3.7	26.45408	17.73903	0.67056
3.8	29.32832	19.67067	0.67071
3.9	32.51202	21.80988	0.67083
4.0	36.03831	24.17894	0.67092
4.1	39.94386	26.80246	0.67100
4.2	44.26924	29.70771	0.67107
4.3	49.05939	32.92486	0.67112
4.4	54.36401	36.48730	0.67117
4.5	60.23811	40.43196	0.67120
4.6	66.74260	44.79975	0.67123
4.7	73.94486	49.63591	0.67126
4.8	81.91947	54.99052	0.67128
4.9	90.74895	60.91899	0.67129
5.0	100.52464	67.48265	0.67130
5.1	111.34759	74.74935	0.67132
5.2	123.32965	82.79418	0.67132
5.3	136.59457	91.70023	0.67133
5.4	151.27930	101.55944	0.67134
5.5	167.53537	112.47355	0.67134
5.6	185.53044	124.55513	0.67135
5.7	205.45005	137.92875	0.67135
5.8	227.49948	152.73222	0.67135
5.9	251.90588	169.11805	0.67135
6.0	278.92057	187.25497	0.67136

x	$F_{2/5}(x)$	$H_{3/5}(x)$	$T_{2/5}(x)$
6.0	278.92057	187.25497	0.67136
6.1	308.82162	207.32967	0.67136
6.2	341.91671	229.54871	0.67136
6.3	378.54625	254.14063	0.67136
6.4	419.08687	281.35829	0.67136
6.5	463.95527	311.48146	0.67136
6.6	513.61248	344.81964	0.67136
6.7	568.56856	381.71529	0.67136
6.8	629.38782	422.54725	0.67136
6.9	696.69459	467.73468	0.67136
7.0	771.17956	517.74129	0.67136
7.1	853.60686	573.08008	0.67136
7.2	944.82186	634.31861	0.67136
7.3	1045.75977	702.08474	0.67136
7.4	1157.45522	777.07309	0.67136
7.5	1281.05275	860.05207	0.67136
7.6	1417.81857	951.87175	0.67136
7.7	1569.15340	1053.47253	0.67136
7.8	1736.60676	1165.89472	0.67136
7.9	1921.89281	1290.28911	0.67136
8.0	2126.90775	1427.92878	0.67136
8.1	2353.74921	1580.22198	0.67136
8.2	2604.73758	1748.72653	0.67136
8.3	2882.43966	1935.16571	0.67136
8.4	3189.69479	2141.44573	0.67136
8.5	3529.64379	2369.67524	0.67136
8.6	3905.76091	2622.18672	0.67136
8.7	4321.88925	2901.56028	0.67136
8.8	4782.27981	3210.64990	0.67136
8.9	5291.63481	3552.61247	0.67136
9.0	5855.15553	3930.93995	0.67136
9.1	6478.59521	4349.49486	0.67136
9.2	7168.31763	4812.54961	0.67136
9.3	7931.36179	5324.82992	0.67136
9.4	8775.51354	5891.56294	0.67136
9.5	9709.38476	6518.53038	0.67136
9.6	10742.50093	7212.12729	0.67136
9.7	11885.39794	7979.42709	0.67136
9.8	13149.72926	8828.25351	0.67136
9.9	14548.38436	9767.26006	0.67136
10.0	16095.61973	10806.01804	0.67136

TABLE 7B. Lanchester-Clifford-Schläfli Functions $F_\alpha(x)$, $H_{1-\alpha}(x)$, and $T_\alpha(x)$ for $\alpha = 2/5$ and x from 1.50 to 10.0.

$\alpha = 3/5$

x	$F_{3/5}(x)$	$H_{2/5}(x)$	$T_{3/5}(x)$
0.0	1.00000	0.0	0.0
0.01	1.00004	0.03607	0.03607
0.02	1.00017	0.06280	0.06279
0.03	1.00038	0.08687	0.08684
0.04	1.00067	0.10937	0.10929
0.05	1.00104	0.13076	0.13063
0.06	1.00150	0.15133	0.15110
0.07	1.00204	0.17123	0.17088
0.08	1.00267	0.19058	0.19008
0.09	1.00338	0.20948	0.20877
0.10	1.00417	0.22798	0.22703
0.11	1.00505	0.24613	0.24490
0.12	1.00601	0.26398	0.26241
0.13	1.00705	0.28157	0.27959
0.14	1.00818	0.29891	0.29648
0.15	1.00939	0.31603	0.31309
0.16	1.01069	0.33296	0.32944
0.17	1.01207	0.34971	0.34554
0.18	1.01353	0.36631	0.36141
0.19	1.01508	0.38275	0.37706
0.20	1.01672	0.39906	0.39250
0.21	1.01844	0.41525	0.40773
0.22	1.02024	0.43132	0.42276
0.23	1.02213	0.44729	0.43761
0.24	1.02411	0.46317	0.45227
0.25	1.02617	0.47897	0.46675
0.26	1.02832	0.49468	0.48106
0.27	1.03055	0.51032	0.49520
0.28	1.03287	0.52590	0.50917
0.29	1.03527	0.54142	0.52298
0.30	1.03776	0.55689	0.53662
0.31	1.04034	0.57231	0.55012
0.32	1.04301	0.58769	0.56345
0.33	1.04576	0.60303	0.57664
0.34	1.04860	0.61834	0.58968
0.35	1.05153	0.63362	0.60257
0.36	1.05455	0.64887	0.61531
0.37	1.05765	0.66411	0.62791
0.38	1.06085	0.67933	0.64037
0.39	1.06413	0.69454	0.65268
0.40	1.06750	0.70974	0.66486
0.41	1.07097	0.72494	0.67690
0.42	1.07452	0.74013	0.68880
0.43	1.07816	0.75532	0.70057
0.44	1.08189	0.77052	0.71220
0.45	1.08572	0.78573	0.72370
0.46	1.08963	0.80095	0.73506
0.47	1.09364	0.81618	0.74630
0.48	1.09774	0.83143	0.75740
0.49	1.10193	0.84670	0.76838
0.50	1.10622	0.86199	0.77922

x	$F_{3/5}(x)$	$H_{2/5}(x)$	$T_{3/5}(x)$
0.50	1.10622	0.86199	0.77922
0.51	1.11060	0.87731	0.78994
0.52	1.11507	0.89265	0.80054
0.53	1.11963	0.90803	0.81100
0.54	1.12429	0.92343	0.82134
0.55	1.12905	0.93887	0.83156
0.56	1.13390	0.95435	0.84166
0.57	1.13885	0.96987	0.85163
0.58	1.14389	0.98544	0.86148
0.59	1.14903	1.00104	0.87121
0.60	1.15427	1.01670	0.88081
0.61	1.15960	1.03240	0.89030
0.62	1.16504	1.04815	0.89968
0.63	1.17057	1.06396	0.90893
0.64	1.17620	1.07983	0.91807
0.65	1.18193	1.09575	0.92709
0.66	1.18776	1.11174	0.93599
0.67	1.19370	1.12778	0.94478
0.68	1.19973	1.14389	0.95346
0.69	1.20587	1.16007	0.96202
0.70	1.21211	1.17632	0.97048
0.71	1.21845	1.19264	0.97882
0.72	1.22489	1.20903	0.98705
0.73	1.23145	1.22550	0.99517
0.74	1.23810	1.24204	1.00318
0.75	1.24486	1.25866	1.01109
0.76	1.25173	1.27537	1.01889
0.77	1.25870	1.29216	1.02658
0.78	1.26579	1.30903	1.03416
0.79	1.27298	1.32599	1.04165
0.80	1.28028	1.34304	1.04903
0.81	1.28769	1.36019	1.05630
0.82	1.29521	1.37742	1.06348
0.83	1.30284	1.39475	1.07055
0.84	1.31058	1.41218	1.07753
0.85	1.31843	1.42971	1.08440
0.86	1.32640	1.44734	1.09118
0.87	1.33448	1.46508	1.09786
0.88	1.34268	1.48292	1.10445
0.89	1.35099	1.50086	1.11094
0.90	1.35942	1.51892	1.11733
0.91	1.36797	1.53709	1.12363
0.92	1.37663	1.55537	1.12984
0.93	1.38541	1.57377	1.13596
0.94	1.39431	1.59229	1.14199
0.95	1.40333	1.61092	1.14792
0.96	1.41248	1.62968	1.15377
0.97	1.42174	1.64856	1.15953
0.98	1.43113	1.66757	1.16521
0.99	1.44064	1.68670	1.17080
1.00	1.45028	1.70597	1.17630

x	$F_{3/5}(x)$	$H_{2/5}(x)$	$T_{3/5}(x)$
1.00	1.45028	1.70597	1.17630
1.01	1.46004	1.72536	1.18172
1.02	1.46993	1.74489	1.18706
1.03	1.47995	1.76456	1.19231
1.04	1.49009	1.78436	1.19748
1.05	1.50037	1.80431	1.20258
1.06	1.51077	1.82439	1.20759
1.07	1.52131	1.84463	1.21253
1.08	1.53198	1.86500	1.21738
1.09	1.54278	1.88553	1.22217
1.10	1.55371	1.90621	1.22687
1.11	1.56479	1.92704	1.23150
1.12	1.57599	1.94802	1.23606
1.13	1.58734	1.96917	1.24055
1.14	1.59882	1.99047	1.24496
1.15	1.61044	2.01193	1.24930
1.16	1.62221	2.03356	1.25357
1.17	1.63411	2.05535	1.25778
1.18	1.64616	2.07731	1.26191
1.19	1.65836	2.09944	1.26598
1.20	1.67069	2.12175	1.26998
1.21	1.68318	2.14423	1.27392
1.22	1.69581	2.16688	1.27779
1.23	1.70859	2.18972	1.28159
1.24	1.72152	2.21274	1.28534
1.25	1.73460	2.23594	1.28902
1.26	1.74784	2.25932	1.29264
1.27	1.76123	2.28290	1.29620
1.28	1.77477	2.30667	1.29970
1.29	1.78847	2.33063	1.30314
1.30	1.80233	2.35479	1.30653
1.31	1.81634	2.37914	1.30985
1.32	1.83052	2.40369	1.31312
1.33	1.84485	2.42845	1.31634
1.34	1.85935	2.45342	1.31950
1.35	1.87402	2.47859	1.32261
1.36	1.88885	2.50397	1.32566
1.37	1.90384	2.52956	1.32866
1.38	1.91901	2.55537	1.33161
1.39	1.93434	2.58140	1.33451
1.40	1.94985	2.60765	1.33736
1.41	1.96553	2.63412	1.34016
1.42	1.98138	2.66082	1.34291
1.43	1.99741	2.68775	1.34562
1.44	2.01361	2.71490	1.34828
1.45	2.03000	2.74229	1.35089
1.46	2.04656	2.76992	1.35345
1.47	2.06331	2.79779	1.35597
1.48	2.08024	2.82589	1.35845
1.49	2.09735	2.85425	1.36088
1.50	2.11465	2.88285	1.36327

TABLE 8A. Lanchester-Clifford-Schläfli Functions $F_\alpha(x)$, $H_{1-\alpha}(x)$, and $T_\alpha(x)$ for $\alpha = 3/5$ and x from 0.00 to 1.50.

$\alpha = 3/5$

x	$F_{3/5}(x)$	$H_{2/5}(x)$	$T_{3/5}(x)$
1.50	2.11465	2.88285	1.36327
1.51	2.13214	2.91169	1.36562
1.52	2.14982	2.94080	1.36792
1.53	2.16770	2.97015	1.37019
1.54	2.18576	2.99977	1.37241
1.55	2.20402	3.02965	1.37460
1.56	2.22248	3.05979	1.37675
1.57	2.24113	3.09020	1.37886
1.58	2.25999	3.12088	1.38093
1.59	2.27904	3.15183	1.38296
1.60	2.29830	3.18306	1.38496
1.61	2.31777	3.21456	1.38692
1.62	2.33744	3.24635	1.38885
1.63	2.35733	3.27843	1.39074
1.64	2.37742	3.31079	1.39260
1.65	2.39773	3.34344	1.39442
1.66	2.41825	3.37639	1.39621
1.67	2.43899	3.40964	1.39797
1.68	2.45995	3.44319	1.39970
1.69	2.48113	3.47704	1.40139
1.70	2.50253	3.51120	1.40306
1.71	2.52416	3.54567	1.40469
1.72	2.54601	3.58045	1.40630
1.73	2.56810	3.61555	1.40787
1.74	2.59041	3.65098	1.40942
1.75	2.61296	3.68672	1.41094
1.76	2.63574	3.72280	1.41243
1.77	2.65876	3.75920	1.41389
1.78	2.68202	3.79594	1.41533
1.79	2.70553	3.83302	1.41674
1.80	2.72927	3.87044	1.41812
1.81	2.75327	3.90820	1.41948
1.82	2.77751	3.94632	1.42081
1.83	2.80200	3.98478	1.42212
1.84	2.82675	4.02361	1.42341
1.85	2.85175	4.06279	1.42467
1.86	2.87701	4.10234	1.42590
1.87	2.90253	4.14225	1.42712
1.88	2.92831	4.18253	1.42831
1.89	2.95436	4.22320	1.42948
1.90	2.98067	4.26423	1.43063
1.91	3.00726	4.30566	1.43175
1.92	3.03412	4.34747	1.43286
1.93	3.06125	4.38967	1.43394
1.94	3.08866	4.43226	1.43501
1.95	3.11636	4.47526	1.43605
1.96	3.14433	4.51865	1.43708
1.97	3.17259	4.56246	1.43809
1.98	3.20114	4.60668	1.43907
1.99	3.22998	4.65131	1.44004
2.00	3.25912	4.69636	1.44099

x	$F_{3/5}(x)$	$H_{2/5}(x)$	$T_{3/5}(x)$
2.0	3.25912	4.69636	1.44099
2.1	3.56725	5.17096	1.44957
2.2	3.90799	5.69257	1.45665
2.3	4.28459	6.26617	1.46249
2.4	4.70063	6.89726	1.46731
2.5	5.16006	7.59185	1.47127
2.6	5.66727	8.35659	1.47453
2.7	6.22710	9.19878	1.47722
2.8	6.84489	10.12649	1.47942
2.9	7.52655	11.14858	1.48123
3.0	8.27860	12.27485	1.48272
3.1	9.10823	13.51608	1.48394
3.2	10.02339	14.88417	1.48494
3.3	11.03285	16.39224	1.48577
3.4	12.14631	18.05476	1.48644
3.5	13.37444	19.88770	1.48699
3.6	14.72906	21.90868	1.48745
3.7	16.22317	24.13713	1.48782
3.8	17.87117	26.59449	1.48812
3.9	19.68890	29.30442	1.48837
4.0	21.69389	32.29303	1.48858
4.1	23.90546	35.58913	1.48874
4.2	26.34495	39.22452	1.48888
4.3	29.03589	43.23429	1.48899
4.4	32.00427	47.65715	1.48909
4.5	35.27878	52.53584	1.48916
4.6	38.89107	57.91754	1.48922
4.7	42.87609	63.85430	1.48928
4.8	47.27239	70.40357	1.48932
4.9	52.12255	77.62878	1.48935
5.0	57.47356	85.59991	1.48938
5.1	63.37728	94.39423	1.48940
5.2	69.89096	104.09703	1.48942
5.3	77.07779	114.80242	1.48944
5.4	85.00753	126.61432	1.48945
5.5	93.75716	139.64742	1.48946
5.6	103.41170	154.02832	1.48947
5.7	114.06496	169.89679	1.48947
5.8	125.82052	187.40709	1.48948
5.9	138.79270	206.72954	1.48948
6.0	153.10773	228.05212	1.48949

x	$F_{3/5}(x)$	$H_{2/5}(x)$	$T_{3/5}(x)$
6.0	153.10773	228.05212	1.48949
6.1	168.90493	251.58239	1.48949
6.2	186.33813	277.54946	1.48949
6.3	205.57712	306.20625	1.48950
6.4	226.80937	337.83198	1.48950
6.5	250.24183	372.73492	1.48950
6.6	276.10298	411.25538	1.48950
6.7	304.64504	453.76907	1.48950
6.8	336.14649	500.69077	1.48950
6.9	370.91477	552.47842	1.48950
7.0	409.28931	609.63759	1.48950
7.1	451.64484	672.72646	1.48950
7.2	498.39511	742.36131	1.48950
7.3	549.99691	819.22253	1.48950
7.4	606.95457	904.06134	1.48950
7.5	669.82494	997.70714	1.48950
7.6	739.22282	1101.07568	1.48950
7.7	815.82701	1215.17807	1.48950
7.8	900.38700	1341.13066	1.48950
7.9	993.73032	1480.16606	1.48950
8.0	1096.77069	1633.64526	1.48950
8.1	1210.51696	1803.07095	1.48950
8.2	1336.08310	1990.10239	1.48950
8.3	1474.69908	2196.57164	1.48950
8.4	1627.72303	2424.50162	1.48951
8.5	1796.65455	2676.12602	1.48951
8.6	1983.14952	2953.91126	1.48951
8.7	2189.03632	3260.58073	1.48951
8.8	2416.33389	3599.14166	1.48951
8.9	2667.27158	3972.91466	1.48951
9.0	2944.31108	4385.56643	1.48951
9.1	3250.17066	4841.14586	1.48951
9.2	3587.85195	5344.12390	1.48951
9.3	3960.66950	5899.43758	1.48951
9.4	4372.28341	6512.53863	1.48951
9.5	4826.73538	7189.44720	1.48951
9.6	5328.48855	7936.81115	1.48951
9.7	5882.47146	8761.97155	1.48951
9.8	6494.12659	9673.03503	1.48951
9.9	7169.46404	10678.95366	1.48951
10.0	7915.12075	11789.61318	1.48951

TABLE 8B. Lanchester-Clifford-Schläfli Functions $F_\alpha(x)$, $H_{1-\alpha}(x)$, and $T_\alpha(x)$ for $\alpha = 3/5$ and x from 1.50 to 10.0.

$\alpha = 4/5$

x	$F_{4/5}(x)$	$H_{1/5}(x)$	$T_{4/5}(x)$	x	$F_{4/5}(x)$	$H_{1/5}(x)$	$T_{4/5}(x)$	x	$F_{4/5}(x)$	$H_{1/5}(x)$	$T_{4/5}(x)$
0.0	1.00000	0.0	0.0	0.50	1.07949	3.02345	2.80081	1.00	1.33486	4.62477	3.46461
0.01	1.00003	0.60057	0.60056	0.51	1.08276	3.05377	2.82035	1.01	1.34206	4.66104	3.47305
0.02	1.00013	0.79251	0.79241	0.52	1.08610	3.08403	2.83955	1.02	1.34936	4.69757	3.48134
0.03	1.00028	0.93215	0.93189	0.53	1.08951	3.11425	2.85840	1.03	1.35674	4.73435	3.48950
0.04	1.00050	1.04599	1.04547	0.54	1.09299	3.14444	2.87693	1.04	1.36422	4.77139	3.49752
0.05	1.00078	1.14386	1.14297	0.55	1.09654	3.17461	2.89513	1.05	1.37179	4.80869	3.50541
0.06	1.00113	1.23068	1.22929	0.56	1.10015	3.20477	2.91302	1.06	1.37946	4.84626	3.51317
0.C7	1.00153	1.30930	1.30730	0.57	1.10384	3.23493	2.93060	1.07	1.38722	4.88410	3.52080
0.C8	1.00200	1.38157	1.37881	C.58	1.10761	3.26509	2.94788	1.08	1.39507	4.92222	3.52830
0.09	1.00253	1.44873	1.44507	0.59	1.11144	3.29526	2.96486	1.09	1.40302	4.96061	3.53567
0.10	1.00313	1.51169	1.50698	0.60	1.11534	3.32546	2.98156	1.10	1.41107	4.99929	3.54292
0.11	1.00378	1.57112	1.56520	0.61	1.11932	3.35569	2.99798	1.11	1.41921	5.03826	3.55005
0.12	1.00450	1.62754	1.62024	0.62	1.12337	3.38596	3.01411	1.12	1.42745	5.07752	3.55706
0.13	1.00529	1.68137	1.67253	0.63	1.12749	3.41627	3.02998	1.13	1.43579	5.11708	3.56395
0.14	1.00613	1.73293	1.72237	0.64	1.13169	3.44664	3.04558	1.14	1.44423	5.15693	3.57072
C.15	1.00704	1.78249	1.77003	0.65	1.13595	3.47706	3.06092	1.15	1.45276	5.19710	3.57739
0.16	1.00801	1.83029	1.81574	0.66	1.14030	3.50755	3.07600	1.16	1.46140	5.23757	3.58393
0.17	1.00905	1.87650	1.85967	C.67	1.14471	3.53812	3.09083	1.17	1.47014	5.27835	3.59037
0.18	1.01015	1.92129	1.90199	0.68	1.14920	3.56876	3.10542	1.18	1.47898	5.31946	3.59670
0.19	1.01131	1.96481	1.94284	0.69	1.15377	3.59949	3.11977	1.19	1.48793	5.36088	3.60292
0.20	1.01253	2.00716	1.98231	0.70	1.15841	3.63032	3.13387	1.20	1.49697	5.40263	3.60903
0.21	1.01382	2.04846	2.02053	0.71	1.16313	3.66124	3.14775	1.21	1.50613	5.44472	3.61505
0.22	1.01518	2.08880	2.05757	0.72	1.16792	3.69226	3.16139	1.22	1.51538	5.48714	3.62095
0.23	1.01659	2.12825	2.09352	C.73	1.17279	3.72340	3.17482	1.23	1.52475	5.52989	3.62676
0.24	1.01807	2.16691	2.12844	0.74	1.17774	3.75465	3.18802	1.24	1.53421	5.57299	3.63247
0.25	1.01962	2.20482	2.16240	0.75	1.18276	3.78603	3.20100	1.25	1.54379	5.61644	3.63808
0.26	1.02122	2.24204	2.19545	0.76	1.18787	3.81753	3.21377	1.26	1.55348	5.66024	3.64360
0.27	1.02290	2.27865	2.22764	C.77	1.19305	3.84916	3.22633	1.27	1.56327	5.70440	3.64902
0.28	1.02463	2.31467	2.25902	C.78	1.19830	3.88093	3.23868	1.28	1.57317	5.74892	3.65435
0.29	1.02644	2.35015	2.28963	0.79	1.20364	3.91284	3.25084	1.29	1.58319	5.79380	3.65958
0.30	1.02830	2.38515	2.31950	0.80	1.20906	3.94491	3.26279	1.30	1.59331	5.83905	3.66473
0.31	1.03023	2.41968	2.34868	0.81	1.21456	3.97712	3.27455	1.31	1.60355	5.88468	3.66978
0.32	1.03223	2.45380	2.37718	0.82	1.22014	4.00950	3.28611	1.32	1.61390	5.93069	3.67475
0.33	1.03429	2.48752	2.40505	0.83	1.22579	4.04204	3.29748	1.33	1.62437	5.97708	3.67964
0.34	1.03642	2.52088	2.43231	0.84	1.23153	4.07474	3.30867	1.34	1.63495	6.02385	3.68444
C.35	1.03861	2.55391	2.45897	0.85	1.23736	4.10762	3.31968	1.35	1.64564	6.07102	3.68915
0.36	1.04087	2.58663	2.48507	0.86	1.24326	4.14068	3.33050	1.36	1.65645	6.11859	3.69379
0.37	1.04319	2.61906	2.51063	C.87	1.24925	4.17392	3.34115	1.37	1.66738	6.16655	3.69834
0.38	1.04558	2.65124	2.53566	0.88	1.25532	4.20735	3.35162	1.38	1.67843	6.21493	3.70281
0.39	1.04804	2.68317	2.56019	C.89	1.26147	4.24097	3.36192	1.39	1.68960	6.26371	3.70721
0.40	1.05056	2.71488	2.58422	0.90	1.26771	4.27478	3.37205	1.40	1.70089	6.31291	3.71153
0.41	1.05315	2.74639	2.60779	0.91	1.27403	4.30880	3.38201	1.41	1.71230	6.36252	3.71577
0.42	1.05580	2.77771	2.63089	0.92	1.28044	4.34303	3.39181	1.42	1.72383	6.41256	3.71994
C.43	1.05853	2.80886	2.65356	0.93	1.28694	4.37746	3.40145	1.43	1.73549	6.46303	3.72404
0.44	1.06132	2.83987	2.67579	0.94	1.29352	4.41211	3.41093	1.44	1.74727	6.51394	3.72806
0.45	1.06418	2.87073	2.69761	C.95	1.30019	4.44698	3.42026	1.45	1.75918	6.56528	3.73202
0.46	1.06710	2.90147	2.71902	C.96	1.30695	4.48207	3.42943	1.46	1.77121	6.61707	3.73590
0.47	1.07010	2.93210	2.74003	0.97	1.31379	4.51739	3.43844	1.47	1.78337	6.66930	3.73972
0.48	1.07316	2.96263	2.76066	0.98	1.32072	4.55295	3.44731	1.48	1.79566	6.72198	3.74347
C.49	1.07629	2.99308	2.78092	0.99	1.32775	4.58874	3.45604	1.49	1.80808	6.77513	3.74715
0.50	1.07949	3.02345	2.80081	1.00	1.33486	4.62477	3.46461	1.50	1.82062	6.82873	3.75077

TABLE 9A. Lanchester-Clifford-Schläfli Functions $F_\alpha(x)$, $H_{1-\alpha}(x)$, and $T_\alpha(x)$ for $\alpha = 4/5$ and x from 0.00 to 1.50.

$\alpha = 4/5$

x	$F_{4/5}(x)$	$H_{1/5}(x)$	$T_{4/5}(x)$
1.50	1.82062	6.82873	3.75077
1.51	1.83330	6.88281	3.75432
1.52	1.84612	6.93736	3.75781
1.53	1.85906	6.99238	3.76124
1.54	1.87214	7.04789	3.76461
1.55	1.88536	7.10388	3.76791
1.56	1.89872	7.16037	3.77116
1.57	1.91221	7.21735	3.77435
1.58	1.92584	7.27484	3.77749
1.59	1.93961	7.33283	3.78057
1.60	1.95353	7.39134	3.78359
1.61	1.96758	7.45037	3.78656
1.62	1.98179	7.50992	3.78947
1.63	1.99613	7.57000	3.79234
1.64	2.01062	7.63062	3.79515
1.65	2.02527	7.69178	3.79791
1.66	2.04006	7.75348	3.80062
1.67	2.05500	7.81573	3.80328
1.68	2.07009	7.87854	3.80590
1.69	2.08533	7.94192	3.80847
1.70	2.10073	8.00586	3.81099
1.71	2.11628	8.07037	3.81346
1.72	2.13200	8.13547	3.81589
1.73	2.14786	8.20115	3.81828
1.74	2.16389	8.26742	3.82062
1.75	2.18008	8.33429	3.82292
1.76	2.19643	8.40176	3.82518
1.77	2.21295	8.46984	3.82740
1.78	2.22963	8.53854	3.82958
1.79	2.24648	8.60786	3.83171
1.80	2.26349	8.67780	3.83381
1.81	2.28068	8.74838	3.83587
1.82	2.29803	8.81960	3.83790
1.83	2.31556	8.89146	3.83988
1.84	2.33326	8.96398	3.84183
1.85	2.35113	9.03716	3.84374
1.86	2.36919	9.11100	3.84562
1.87	2.38742	9.18551	3.84747
1.88	2.40583	9.26070	3.84928
1.89	2.42442	9.33658	3.85105
1.90	2.44320	9.41315	3.85280
1.91	2.46216	9.49041	3.85451
1.92	2.48131	9.56839	3.85619
1.93	2.50064	9.64707	3.85784
1.94	2.52017	9.72647	3.85946
1.95	2.53988	9.80660	3.86104
1.96	2.55979	9.88746	3.86260
1.97	2.57990	9.96906	3.86413
1.98	2.60020	10.05141	3.86564
1.99	2.62070	10.13452	3.86711
2.00	2.64139	10.21838	3.86856

x	$F_{4/5}(x)$	$H_{1/5}(x)$	$T_{4/5}(x)$
2.0	2.64139	10.21838	3.86856
2.1	2.85978	11.10058	3.88162
2.2	3.10023	12.06747	3.89244
2.3	3.36483	13.12750	3.90139
2.4	3.65585	14.28993	3.90878
2.5	3.97583	15.56493	3.91488
2.6	4.32755	16.96362	3.91992
2.7	4.71406	18.49825	3.92406
2.8	5.13873	20.18227	3.92748
2.9	5.60529	22.03044	3.93029
3.0	6.11783	24.05901	3.93261
3.1	6.68084	26.28582	3.93451
3.2	7.29929	28.73051	3.93607
3.3	7.97862	31.41467	3.93736
3.4	8.72485	34.36206	3.93841
3.5	9.54459	37.59880	3.93928
3.6	10.44511	41.15365	3.93999
3.7	11.43442	45.05822	3.94058
3.8	12.52134	49.34733	3.94106
3.9	13.71558	54.05928	3.94145
4.0	15.02781	59.23621	3.94177
4.1	16.46979	64.92454	3.94204
4.2	18.05446	71.17530	3.94226
4.3	19.79607	78.04472	3.94243
4.4	21.71031	85.59465	3.94258
4.5	23.81444	93.89320	3.94270
4.6	26.12747	103.01536	3.94280
4.7	28.67033	113.04365	3.94288
4.8	31.46606	124.06897	3.94295
4.9	34.54004	136.19136	3.94300
5.0	37.92019	149.52099	3.94304
5.1	41.63729	164.17919	3.94308
5.2	45.72521	180.29953	3.94311
5.3	50.22124	198.02911	3.94313
5.4	55.16647	217.52993	3.94315
5.5	60.60614	238.98039	3.94317
5.6	66.59008	262.57698	3.94318
5.7	73.17316	288.53610	3.94320
5.8	80.41583	317.09608	3.94320
5.9	88.38466	348.51946	3.94321
6.0	97.15298	383.09540	3.94322

x	$F_{4/5}(x)$	$H_{1/5}(x)$	$T_{4/5}(x)$
6.0	97.15298	383.09540	3.94322
6.1	106.80157	421.14242	3.94322
6.2	117.41941	463.01142	3.94323
6.3	129.10452	509.08890	3.94323
6.4	141.96489	559.80068	3.94323
6.5	156.11948	615.61587	3.94324
6.6	171.69934	677.05126	3.94324
6.7	188.84889	744.67626	3.94324
6.8	207.72720	819.11820	3.94324
6.9	228.50956	901.06828	3.94324
7.0	251.38912	991.28809	3.94324
7.1	276.57868	1090.61681	3.94324
7.2	304.31274	1199.97911	3.94324
7.3	334.84970	1320.39393	3.94324
7.4	368.47431	1452.98408	3.94324
7.5	405.50035	1598.98691	3.94324
7.6	446.27361	1759.76597	3.94324
7.7	491.17519	1936.82395	3.94324
7.8	540.62505	2131.81696	3.94324
7.9	595.08607	2346.57016	3.94324
8.0	655.06836	2583.09512	3.94325
8.1	721.13418	2843.60889	3.94325
8.2	793.90321	3130.55508	3.94325
8.3	874.05848	3446.62701	3.94325
8.4	962.35283	3794.79337	3.94325
8.5	1059.61611	4178.32639	3.94325
8.6	1166.76302	4600.83300	3.94325
8.7	1284.80185	5066.28910	3.94325
8.8	1414.84405	5579.07745	3.94325
8.9	1558.11484	6144.02939	3.94325
9.0	1715.96487	6766.47083	3.94325
9.1	1889.88308	7452.27307	3.94325
9.2	2081.51091	8207.90869	3.94325
9.3	2292.65795	9040.51334	3.94325
9.4	2525.31920	9957.95379	3.94325
9.5	2781.69411	10968.90301	3.94325
9.6	3064.20757	12082.92298	3.94325
9.7	3375.53310	13310.25601	3.94325
9.8	3718.61835	14663.42546	3.94325
9.9	4096.71333	16154.34682	3.94325
10.0	4513.40140	17797.45024	3.94325

TABLE 9B. Lanchester-Clifford-Schläfli Functions $F_\alpha(x)$, $H_{1-\alpha}(x)$, and $T_\alpha(x)$ for $\alpha = 4/5$ and x from 1.50 to 10.0.

$\alpha = 3/7$

x	$F_{3/7}(x)$	$H_{4/7}(x)$	$T_{3/7}(x)$
0.0	1.00000	0.0	0.0
0.01	1.00006	0.00410	0.00410
0.02	1.00023	0.00906	0.00906
0.03	1.00053	0.01441	0.01440
0.04	1.00093	0.02002	0.02000
0.05	1.00146	0.02584	0.02580
0.06	1.00210	0.03183	0.03176
0.07	1.00286	0.03797	0.03786
0.08	1.00374	0.04424	0.04408
0.09	1.00473	0.05063	0.05039
0.10	1.00584	0.05713	0.05679
0.11	1.00707	0.06372	0.06327
0.12	1.00841	0.07041	0.06982
0.13	1.00987	0.07719	0.07643
0.14	1.01145	0.08404	0.08309
0.15	1.01315	0.09098	0.08980
0.16	1.01497	0.09799	0.09655
0.17	1.01690	0.10508	0.10333
0.18	1.01895	0.11223	0.11015
0.19	1.02112	0.11946	0.11699
0.20	1.02342	0.12675	0.12385
0.21	1.02582	0.13410	0.13073
0.22	1.02835	0.14152	0.13762
0.23	1.03100	0.14900	0.14452
0.24	1.03377	0.15655	0.15143
0.25	1.03666	0.16415	0.15835
0.26	1.03967	0.17182	0.16526
0.27	1.04280	0.17954	0.17217
0.28	1.04605	0.18732	0.17908
0.29	1.04942	0.19516	0.18597
0.30	1.05291	0.20306	0.19286
0.31	1.05653	0.21102	0.19973
0.32	1.06027	0.21904	0.20658
0.33	1.06413	0.22711	0.21342
0.34	1.06812	0.23524	0.22024
0.35	1.07223	0.24343	0.22703
0.36	1.07646	0.25167	0.23380
0.37	1.08082	0.25998	0.24054
0.38	1.08530	0.26834	0.24725
0.39	1.08991	0.27676	0.25393
0.40	1.09465	0.28524	0.26058
0.41	1.09951	0.29378	0.26719
0.42	1.10450	0.30238	0.27377
0.43	1.10961	0.31104	0.28031
0.44	1.11486	0.31976	0.28681
0.45	1.12023	0.32853	0.29327
0.46	1.12574	0.33737	0.29969
0.47	1.13137	0.34627	0.30607
0.48	1.13713	0.35523	0.31240
0.49	1.14303	0.36426	0.31868
0.50	1.14905	0.37335	0.32492

x	$F_{3/7}(x)$	$H_{4/7}(x)$	$T_{3/7}(x)$
0.50	1.14905	0.37335	0.32492
0.51	1.15521	0.38250	0.33111
0.52	1.16150	0.39171	0.33725
0.53	1.16792	0.40099	0.34333
0.54	1.17448	0.41033	0.34937
0.55	1.16118	0.41974	0.35536
0.56	1.18801	0.42922	0.36129
0.57	1.19497	0.43876	0.36717
0.58	1.20208	0.44837	0.37300
0.59	1.20932	0.45805	0.37877
0.60	1.21670	0.46780	0.38449
0.61	1.22422	0.47762	0.39015
0.62	1.23188	0.48751	0.39575
0.63	1.23968	0.49748	0.40130
0.64	1.24762	0.50751	0.40678
0.65	1.25570	0.51762	0.41221
0.66	1.26393	0.52780	0.41759
0.67	1.27230	0.53806	0.42290
0.68	1.28082	0.54839	0.42815
0.69	1.28949	0.55880	0.43335
0.70	1.29830	0.56928	0.43849
0.71	1.30726	0.57985	0.44356
0.72	1.31636	0.59049	0.44858
0.73	1.32562	0.60122	0.45354
0.74	1.33503	0.61202	0.45843
0.75	1.34459	0.62291	0.46327
0.76	1.35430	0.63388	0.46805
0.77	1.36417	0.64494	0.47277
0.78	1.37419	0.65608	0.47743
0.79	1.38437	0.66730	0.48203
0.80	1.39470	0.67862	0.48657
0.81	1.40520	0.69002	0.49105
0.82	1.41585	0.70151	0.49547
0.83	1.42666	0.71309	0.49983
0.84	1.43764	0.72476	0.50414
0.85	1.44877	0.73653	0.50838
0.86	1.46007	0.74839	0.51257
0.87	1.47154	0.76034	0.51670
0.88	1.48317	0.77239	0.52077
0.89	1.49497	0.78453	0.52478
0.90	1.50693	0.79678	0.52874
0.91	1.51907	0.80912	0.53264
0.92	1.53138	0.82156	0.53648
0.93	1.54386	0.83411	0.54027
0.94	1.55651	0.84675	0.54401
0.95	1.56934	0.85950	0.54768
0.96	1.58235	0.87236	0.55131
0.97	1.59553	0.88532	0.55488
0.98	1.60889	0.89839	0.55839
0.99	1.62243	0.91157	0.56186
1.00	1.63616	0.92486	0.56526

x	$F_{3/7}(x)$	$H_{4/7}(x)$	$T_{3/7}(x)$
1.00	1.63616	0.92486	0.56526
1.01	1.65007	0.93826	0.56862
1.02	1.66416	0.95178	0.57193
1.03	1.67844	0.96541	0.57518
1.04	1.69291	0.97915	0.57838
1.05	1.70756	0.99301	0.58154
1.06	1.72241	1.00699	0.58464
1.07	1.73745	1.02109	0.58770
1.08	1.75268	1.03531	0.59070
1.09	1.76811	1.04965	0.59366
1.10	1.78374	1.06412	0.59657
1.11	1.79956	1.07871	0.59943
1.12	1.81559	1.09343	0.60224
1.13	1.83181	1.10827	0.60501
1.14	1.84824	1.12325	0.60774
1.15	1.86488	1.13836	0.61042
1.16	1.88172	1.15360	0.61305
1.17	1.89878	1.16897	0.61564
1.18	1.91604	1.18448	0.61819
1.19	1.93352	1.20013	0.62070
1.20	1.95121	1.21591	0.62316
1.21	1.96912	1.23184	0.62558
1.22	1.98724	1.24791	0.62796
1.23	2.00559	1.26412	0.63030
1.24	2.02416	1.28048	0.63260
1.25	2.04295	1.29698	0.63486
1.26	2.06196	1.31363	0.63708
1.27	2.08121	1.33044	0.63926
1.28	2.10068	1.34739	0.64141
1.29	2.12039	1.36450	0.64351
1.30	2.14033	1.38177	0.64558
1.31	2.16051	1.39919	0.64762
1.32	2.18093	1.41677	0.64962
1.33	2.20158	1.43451	0.65158
1.34	2.22248	1.45241	0.65351
1.35	2.24362	1.47048	0.65541
1.36	2.26501	1.48872	0.65727
1.37	2.28665	1.50712	0.65910
1.38	2.30854	1.52569	0.66089
1.39	2.33068	1.54444	0.66265
1.40	2.35308	1.56336	0.66439
1.41	2.37574	1.58245	0.66609
1.42	2.39866	1.60172	0.66776
1.43	2.42184	1.62118	0.66940
1.44	2.44529	1.64081	0.67101
1.45	2.46901	1.66063	0.67259
1.46	2.49299	1.68063	0.67414
1.47	2.51725	1.70082	0.67567
1.48	2.54178	1.72120	0.67716
1.49	2.56660	1.74177	0.67863
1.50	2.59169	1.76254	0.68007

TABLE 10A. Lanchester-Clifford-Schläfli Functions $F_\alpha(x)$, $H_{1-\alpha}(x)$, and $T_\alpha(x)$ for $\alpha = 3/7$ and x from 0.00 to 1.50.

$\alpha = 3/7$

x	$F_{3/7}(x)$	$H_{4/7}(x)$	$T_{3/7}(x)$
1.50	2.59169	1.76254	0.68007
1.51	2.61706	1.78350	0.68149
1.52	2.64273	1.80466	0.68288
1.53	2.66868	1.82602	0.68424
1.54	2.69492	1.84759	0.68558
1.55	2.72145	1.86936	0.68690
1.56	2.74828	1.89133	0.68819
1.57	2.77542	1.91352	0.68945
1.58	2.80285	1.93591	0.69069
1.59	2.83059	1.95852	0.69191
1.60	2.85864	1.98135	0.69311
1.61	2.88699	2.00440	0.69428
1.62	2.91567	2.02766	0.69544
1.63	2.94466	2.05115	0.69657
1.64	2.97397	2.07487	0.69768
1.65	3.00360	2.09881	0.69877
1.66	3.03356	2.12299	0.69983
1.67	3.06384	2.14739	0.70088
1.68	3.09446	2.17204	0.70191
1.69	3.12542	2.19692	0.70292
1.70	3.15672	2.22204	0.70391
1.71	3.18835	2.24741	0.70488
1.72	3.22033	2.27302	0.70583
1.73	3.25267	2.29888	0.70677
1.74	3.28535	2.32499	0.70769
1.75	3.31839	2.35136	0.70858
1.76	3.35178	2.37798	0.70947
1.77	3.38555	2.40486	0.71033
1.78	3.41967	2.43201	0.71118
1.79	3.45417	2.45942	0.71201
1.80	3.48904	2.48710	0.71283
1.81	3.52428	2.51504	0.71363
1.82	3.55991	2.54327	0.71442
1.83	3.59592	2.57177	0.71519
1.84	3.63231	2.60054	0.71595
1.85	3.66910	2.62960	0.71669
1.86	3.70629	2.65895	0.71742
1.87	3.74387	2.68859	0.71813
1.88	3.78186	2.71851	0.71883
1.89	3.82025	2.74873	0.71952
1.90	3.85906	2.77925	0.72019
1.91	3.89828	2.81007	0.72085
1.92	3.93792	2.84120	0.72150
1.93	3.97799	2.87263	0.72213
1.94	4.01848	2.90437	0.72275
1.95	4.05940	2.93643	0.72336
1.96	4.10076	2.96880	0.72396
1.97	4.14256	3.00149	0.72455
1.98	4.18480	3.03451	0.72513
1.99	4.22750	3.06786	0.72569
2.00	4.27065	3.10153	0.72624

x	$F_{3/7}(x)$	$H_{4/7}(x)$	$T_{3/7}(x)$
2.0	4.27065	3.10153	0.72624
2.1	4.72807	3.45728	0.73122
2.2	5.23611	3.85017	0.73531
2.3	5.80002	4.28427	0.73866
2.4	6.42568	4.76410	0.74142
2.5	7.11957	5.29460	0.74367
2.6	7.88889	5.88129	0.74552
2.7	8.74163	6.53024	0.74703
2.8	9.68661	7.24815	0.74827
2.9	10.73361	8.04247	0.74928
3.0	11.6[illegible]7	8.92139	0.75011
3.1	13.17819	9.89402	0.75079
3.2	14.60103	10.97038	0.75134
3.3	16.17669	12.16159	0.75180
3.4	17.92145	13.47996	0.75217
3.5	19.85328	14.93907	0.75247
3.6	21.99212	16.55396	0.75272
3.7	24.36000	18.34129	0.75293
3.8	26.98131	20.31944	0.75309
3.9	29.88304	22.50879	0.75323
4.0	33.09505	24.93186	0.75334
4.1	36.65035	27.61356	0.75343
4.2	40.58552	30.58148	0.75351
4.3	44.94097	33.86609	0.75357
4.4	49.76144	37.50113	0.75362
4.5	55.09643	41.52393	0.75366
4.6	61.00067	45.97576	0.75369
4.7	67.53473	50.90229	0.75372
4.8	74.76562	56.35404	0.75374
4.9	82.76745	62.38688	0.75376
5.0	91.62221	69.06263	0.75378
5.1	101.42059	76.44966	0.75379
5.2	112.26292	84.62362	0.75380
5.3	124.26019	93.66816	0.75381
5.4	137.53514	103.67584	0.75381
5.5	152.22357	114.74901	0.75382
5.6	168.47567	127.00092	0.75382
5.7	186.45759	140.55680	0.75383
5.8	206.35310	155.55520	0.75383
5.9	228.36545	172.14934	0.75383
6.0	252.71948	190.50872	0.75383

x	$F_{3/7}(x)$	$H_{4/7}(x)$	$T_{3/7}(x)$
6.0	252.71948	190.50872	0.75383
6.1	279.66387	210.82079	0.75384
6.2	309.47366	233.29292	0.75384
6.3	342.45309	258.15445	0.75384
6.4	378.93865	285.65902	0.75384
6.5	419.30250	316.08721	0.75384
6.6	463.95628	349.74932	0.75384
6.7	513.35525	386.98855	0.75384
6.8	568.00295	428.18449	0.75384
6.9	628.45630	473.75698	0.75384
7.0	695.33122	524.17032	0.75384
7.1	769.30895	579.93806	0.75384
7.2	851.14290	641.62815	0.75384
7.3	941.66633	709.86875	0.75384
7.4	1041.80078	785.35455	0.75384
7.5	1152.56546	868.85388	0.75384
7.6	1275.08753	961.21644	0.75384
7.7	1410.61358	1063.38196	0.75384
7.8	1560.52225	1176.38974	0.75384
7.9	1726.33822	1301.38913	0.75384
8.0	1909.74766	1439.65124	0.75384
8.1	2112.61531	1592.58180	0.75384
8.2	2337.00342	1761.73543	0.75384
8.3	2585.19263	1948.83137	0.75384
8.4	2859.70505	2155.77090	0.75384
8.5	3163.32988	2384.65663	0.75384
8.6	3499.15164	2637.81382	0.75384
8.7	3870.58138	2917.81387	0.75384
8.8	4281.39132	3227.50044	0.75384
8.9	4735.75299	3570.01821	0.75384
9.0	5238.27950	3948.84475	0.75384
9.1	5794.07225	4367.82572	0.75384
9.2	6408.77262	4831.21383	0.75384
9.3	7088.61905	5343.71190	0.75384
9.4	7840.51021	5910.52045	0.75384
9.5	8672.07486	6537.39037	0.75384
9.6	9591.74906	7230.68112	0.75384
9.7	10608.86152	7997.42512	0.75384
9.8	11733.72807	8845.39888	0.75384
9.9	12977.75603	9783.20173	0.75384
10.0	14353.55964	10820.34285	0.75384

TABLE 10B. Lanchester-Clifford-Schläfli Functions $F_\alpha(x)$, $H_{1-\alpha}(x)$, and $T_\alpha(x)$ for $\alpha = 3/7$ and x from 1.50 to 10.0.

$\alpha = 4/7$

x	$F_{4/7}(x)$	$F_{3/7}(x)$	$T_{4/7}(x)$	x	$F_{4/7}(x)$	$H_{3/7}(x)$	$T_{4/7}(x)$	x	$F_{4/7}(x)$	$H_{3/7}(x)$	$T_{4/7}(x)$
0.0	1.00000	0.0	0.0	0.50	1.11157	0.74260	0.66807	1.00	1.47345	1.52541	1.03527
0.01	1.00004	0.02487	0.02487	0.51	1.11617	0.75663	0.67788	1.01	1.48373	1.54354	1.04031
0.02	1.00018	0.04505	0.04504	0.52	1.12087	0.77069	0.68758	1.02	1.49414	1.56180	1.04528
0.03	1.00039	0.06378	0.06376	0.53	1.12566	0.78479	0.69718	1.03	1.50468	1.58018	1.05018
0.04	1.00070	0.08163	0.08157	0.54	1.13056	0.79893	0.70666	1.04	1.51537	1.59870	1.05499
0.05	1.00109	0.09885	0.09874	0.55	1.13556	0.81311	0.71604	1.05	1.52619	1.61736	1.05974
0.06	1.00158	0.11559	0.11541	0.56	1.14066	0.82734	0.72532	1.06	1.53714	1.63615	1.06441
0.07	1.00214	0.13195	0.13167	0.57	1.14586	0.84161	0.73448	1.07	1.54824	1.65508	1.06901
0.08	1.00280	0.14799	0.14758	0.58	1.15116	0.85593	0.74354	1.08	1.55947	1.67415	1.07353
0.09	1.00355	0.16376	0.16318	0.59	1.15656	0.87030	0.75249	1.09	1.57085	1.69336	1.07799
0.10	1.00438	0.17929	0.17851	0.60	1.16206	0.88473	0.76134	1.10	1.58237	1.71271	1.08237
0.11	1.00530	0.19463	0.19360	0.61	1.16767	0.89920	0.77008	1.11	1.59403	1.73221	1.08669
0.12	1.00631	0.20978	0.20847	0.62	1.17338	0.91374	0.77872	1.12	1.60584	1.75186	1.09093
0.13	1.00740	0.22478	0.22313	0.63	1.17920	0.92833	0.78726	1.13	1.61779	1.77166	1.09511
0.14	1.00859	0.23963	0.23759	0.64	1.18512	0.94298	0.79569	1.14	1.62989	1.79161	1.09923
0.15	1.00986	0.25436	0.25188	0.65	1.19114	0.95769	0.80401	1.15	1.64213	1.81172	1.10327
0.16	1.01122	0.26898	0.26599	0.66	1.19727	0.97247	0.81224	1.16	1.65453	1.83198	1.10725
0.17	1.01267	0.28348	0.27994	0.67	1.20351	0.98731	0.82036	1.17	1.66707	1.85240	1.11117
0.18	1.01421	0.29790	0.29373	0.68	1.20985	1.00222	0.82838	1.18	1.67977	1.87298	1.11502
0.19	1.01584	0.31223	0.30737	0.69	1.21630	1.01720	0.83630	1.19	1.69262	1.89372	1.11881
0.20	1.01756	0.32649	0.32086	0.70	1.22286	1.03224	0.84412	1.20	1.70562	1.91463	1.12254
0.21	1.01936	0.34068	0.33421	0.71	1.22953	1.04736	0.85184	1.21	1.71877	1.93570	1.12621
0.22	1.02126	0.35480	0.34742	0.72	1.23631	1.06256	0.85946	1.22	1.73209	1.95694	1.12982
0.23	1.02324	0.36887	0.36049	0.73	1.24320	1.07783	0.86698	1.23	1.74556	1.97835	1.13336
0.24	1.02532	0.38289	0.37343	0.74	1.25020	1.09318	0.87440	1.24	1.75919	1.99994	1.13685
0.25	1.02748	0.39686	0.38625	0.75	1.25731	1.10861	0.88173	1.25	1.77298	2.02170	1.14028
0.26	1.02973	0.41079	0.39893	0.76	1.26453	1.12411	0.88896	1.26	1.78693	2.04364	1.14366
0.27	1.03208	0.42469	0.41149	0.77	1.27186	1.13971	0.89609	1.27	1.80105	2.06575	1.14697
0.28	1.03451	0.43856	0.42393	0.78	1.27931	1.15538	0.90313	1.28	1.81532	2.08805	1.15024
0.29	1.03704	0.45240	0.43624	0.79	1.28688	1.17115	0.91007	1.29	1.82977	2.11053	1.15344
0.30	1.03966	0.46622	0.44843	0.80	1.29455	1.18700	0.91692	1.30	1.84438	2.13320	1.15660
0.31	1.04237	0.48001	0.46050	0.81	1.30235	1.20294	0.92367	1.31	1.85916	2.15606	1.15970
0.32	1.04517	0.49380	0.47246	0.82	1.31026	1.21898	0.93034	1.32	1.87411	2.17910	1.16274
0.33	1.04806	0.50757	0.48429	0.83	1.31828	1.23511	0.93691	1.33	1.88923	2.20234	1.16574
0.34	1.05104	0.52133	0.49601	0.84	1.32643	1.25133	0.94339	1.34	1.90452	2.22578	1.16868
0.35	1.05412	0.53508	0.50761	0.85	1.33469	1.26766	0.94977	1.35	1.91999	2.24941	1.17158
0.36	1.05729	0.54884	0.51910	0.86	1.34307	1.28408	0.95607	1.36	1.93563	2.27324	1.17442
0.37	1.06055	0.56259	0.53047	0.87	1.35158	1.30060	0.96228	1.37	1.95145	2.29728	1.17722
0.38	1.06390	0.57635	0.54173	0.88	1.36020	1.31722	0.96840	1.38	1.96744	2.32152	1.17997
0.39	1.06735	0.59011	0.55287	0.89	1.36895	1.33395	0.97444	1.39	1.98362	2.34596	1.18267
0.40	1.07090	0.60388	0.56390	0.90	1.37782	1.35079	0.98039	1.40	1.99998	2.37062	1.18532
0.41	1.07453	0.61766	0.57482	0.91	1.38681	1.36774	0.98625	1.41	2.01652	2.39549	1.18793
0.42	1.07826	0.63146	0.58562	0.92	1.39593	1.38479	0.99202	1.42	2.03325	2.42057	1.19049
0.43	1.08209	0.64527	0.59632	0.93	1.40517	1.40196	0.99771	1.43	2.05017	2.44587	1.19301
0.44	1.08601	0.65910	0.60690	0.94	1.41454	1.41924	1.00332	1.44	2.06727	2.47138	1.19548
0.45	1.09003	0.67295	0.61737	0.95	1.42403	1.43663	1.00885	1.45	2.08456	2.49712	1.19791
0.46	1.09414	0.68683	0.62773	0.96	1.43366	1.45415	1.01429	1.46	2.10204	2.52308	1.20030
0.47	1.09835	0.70073	0.63798	0.97	1.44341	1.47178	1.01966	1.47	2.11972	2.54928	1.20265
0.48	1.10266	0.71466	0.64812	0.98	1.45329	1.48953	1.02494	1.48	2.13759	2.57569	1.20495
0.49	1.10707	0.72861	0.65815	0.99	1.46330	1.50741	1.03014	1.49	2.15566	2.60235	1.20722
0.50	1.11157	0.74260	0.66807	1.00	1.47345	1.52541	1.03527	1.50	2.17392	2.62923	1.20944

TABLE 11A. Lanchester-Clifford-Schläfli Functions $F_\alpha(x)$, $H_{1-\alpha}(x)$, and $T_\alpha(x)$ for $\alpha = 4/7$ and x from 0.00 to 1.50.

$\alpha = 4/7$

x	$F_{4/7}(x)$	$H_{3/7}(x)$	$T_{4/7}(x)$
1.50	2.17392	2.62923	1.20944
1.51	2.19239	2.65636	1.21163
1.52	2.21105	2.68372	1.21377
1.53	2.22992	2.71132	1.21588
1.54	2.24900	2.73917	1.21795
1.55	2.26828	2.76727	1.21999
1.56	2.28777	2.79562	1.22198
1.57	2.30747	2.82423	1.22395
1.58	2.32739	2.85309	1.22587
1.59	2.34752	2.88220	1.22777
1.60	2.36787	2.91158	1.22962
1.61	2.38843	2.94123	1.23145
1.62	2.40922	2.97114	1.23324
1.63	2.43022	3.00133	1.23500
1.64	2.45146	3.03178	1.23673
1.65	2.47292	3.06252	1.23842
1.66	2.49460	3.09353	1.24009
1.67	2.51652	3.12483	1.24172
1.68	2.53867	3.15641	1.24333
1.69	2.56106	3.18828	1.24491
1.70	2.58368	3.22044	1.24645
1.71	2.60654	3.25289	1.24797
1.72	2.62965	3.28565	1.24946
1.73	2.65300	3.31871	1.25093
1.74	2.67659	3.35207	1.25237
1.75	2.70043	3.38573	1.25378
1.76	2.72452	3.41971	1.25516
1.77	2.74887	3.45401	1.25652
1.78	2.77347	3.48862	1.25785
1.79	2.79832	3.52355	1.25916
1.80	2.82344	3.55880	1.26045
1.81	2.84882	3.59439	1.26171
1.82	2.87447	3.63030	1.26295
1.83	2.90038	3.66655	1.26416
1.84	2.92656	3.70314	1.26536
1.85	2.95301	3.74007	1.26653
1.86	2.97974	3.77735	1.26768
1.87	3.00675	3.81497	1.26880
1.88	3.03403	3.85295	1.26991
1.89	3.06160	3.89128	1.27100
1.90	3.08945	3.92997	1.27206
1.91	3.11760	3.96903	1.27311
1.92	3.14603	4.00846	1.27413
1.93	3.17475	4.04825	1.27514
1.94	3.20377	4.08842	1.27613
1.95	3.23309	4.12897	1.27710
1.96	3.26272	4.16991	1.27805
1.97	3.29264	4.21123	1.27898
1.98	3.32288	4.25294	1.27990
1.99	3.35342	4.29504	1.28079
2.00	3.38428	4.33755	1.28168

x	$F_{4/7}(x)$	$H_{3/7}(x)$	$T_{4/7}(x)$
2.0	3.38428	4.33755	1.28168
2.1	3.71076	4.78548	1.28962
2.2	4.07203	5.27811	1.29619
2.3	4.47160	5.82021	1.30159
2.4	4.91330	6.41702	1.30605
2.5	5.40141	7.07432	1.30972
2.6	5.94065	7.79848	1.31273
2.7	6.53623	8.59651	1.31521
2.8	7.19391	9.47614	1.31725
2.9	7.92006	10.44590	1.31892
3.0	8.72171	11.51518	1.32029
3.1	9.60664	12.69435	1.32141
3.2	10.58343	13.99486	1.32234
3.3	11.66155	15.42934	1.32309
3.4	12.85148	17.01171	1.32372
3.5	14.16478	18.75735	1.32422
3.6	15.61420	20.68323	1.32464
3.7	17.21385	22.80807	1.32498
3.8	18.97929	25.15257	1.32526
3.9	20.92769	27.73953	1.32549
4.0	23.07805	30.59416	1.32568
4.1	25.45130	33.74426	1.32584
4.2	28.07058	37.22054	1.32596
4.3	30.96144	41.05691	1.32607
4.4	34.15206	45.29079	1.32615
4.5	37.67360	49.96350	1.32622
4.6	41.56043	55.12066	1.32628
4.7	45.85052	60.81264	1.32632
4.8	50.58579	67.09506	1.32636
4.9	55.81251	74.02933	1.32639
5.0	61.58180	81.68325	1.32642
5.1	67.95008	90.13169	1.32644
5.2	74.97967	99.45731	1.32646
5.3	82.73938	109.75138	1.32647
5.4	91.30517	121.11471	1.32648
5.5	100.76094	133.65857	1.32649
5.6	111.19932	147.50587	1.32650
5.7	122.72263	162.79230	1.32651
5.8	135.44380	179.66772	1.32651
5.9	149.48759	198.29760	1.32652
6.0	164.99174	218.86467	1.32652

x	$F_{4/7}(x)$	$H_{3/7}(x)$	$T_{4/7}(x)$
6.0	164.99174	218.86467	1.32652
6.1	182.10835	241.57070	1.32652
6.2	201.00541	266.63852	1.32652
6.3	221.86839	294.31420	1.32653
6.4	244.90215	324.86946	1.32653
6.5	270.33290	358.60439	1.32653
6.6	298.41044	395.85037	1.32653
6.7	329.41064	436.97335	1.32653
6.8	363.63818	482.37748	1.32653
6.9	401.42949	532.50907	1.32653
7.0	443.15614	587.86102	1.32653
7.1	489.22847	648.97766	1.32653
7.2	540.09965	716.46013	1.32653
7.3	596.27016	790.97233	1.32653
7.4	658.29272	873.24746	1.32653
7.5	726.77774	964.09527	1.32653
7.6	802.39937	1064.41005	1.32653
7.7	885.90216	1175.17944	1.32653
7.8	578.10839	1297.49424	1.32653
7.9	1079.92621	1432.55915	1.32653
8.0	1192.35862	1581.70466	1.32653
8.1	1316.51334	1746.40024	1.32653
8.2	1453.61383	1928.26881	1.32653
8.3	1605.01131	2129.10284	1.32653
8.4	1772.19815	2350.88200	1.32653
8.5	1956.82264	2595.79281	1.32653
8.6	2160.70525	2866.25017	1.32653
8.7	2385.85663	3164.92129	1.32653
8.8	2634.49754	3494.75209	1.32653
8.9	2909.08076	3858.99625	1.32653
9.0	3212.31536	4261.24747	1.32653
9.1	3547.19354	4705.47498	1.32653
9.2	3917.02016	5196.06282	1.32653
9.3	4325.44549	5737.85318	1.32653
9.4	4776.50131	6336.19435	1.32653
9.5	5274.64079	6996.99364	1.32653
9.6	5824.78258	7726.77579	1.32653
9.7	6432.35948	8532.74759	1.32653
9.8	7103.37220	9422.86922	1.32653
9.9	7844.44879	10405.93299	1.32653
10.0	8662.91020	11491.65041	1.32653

TABLE 11B. Lanchester-Clifford-Schläfli Functions $F_\alpha(x)$, $H_{1-\alpha}(x)$, and $T_\alpha(x)$ for $\alpha = 4/7$ and x from 1.50 to 10.0.